EFFECT OF GRAIN SIZE ON MECHANICAL BEHAVIOUR OF SANDS

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This work is dedicated in memory of the researchers who spent their lives for the development of geotechnical engineereing.

CERTIFICATE

This is to certify that the thesis entitled, 'EFFECT OF GRAIN SIZE ON MECHANICAL BEHAVIOUR OF SANDS', by Mr. Joyis Thomas, for the award of the Master of Technology, Indian Institute of Technology, Kanpur has been carried out under my guidance and that his work has not been submitted elsewhere for the award of any degree.

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Finally, the author would like to thank all those people who made his stay in this institute a memorable one.

-Joyis Thomas

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LIST OF SYMBOLS

e_{max}	-	Maximum void ratio
e_{min}	-	Minimum void ratio
e	-	Void ratio
G	-	Specific gravity
Cu	-	Coefficient of uniformity
D_{10}	-	Effective particle size
D_{50}	-	Average particle size
h	-	Height of fall
ϕ'	-	Effective angle of internal friction
I_D	-	Relative density
I_{DC}	-	Relative density after consolidation
I_{DF}	-	Relative density at failure
M	-	Constrained modulus
τ	-	Peak shear stress
σ'	-	Effective normal stress/confining pressure
$\bar{\sigma}$	-	Mean normal stress
m_o, m_1, m_2	-	Constants involved in modulus formula
α, β^*, γ	-	Constants involved in shear stress formula
q_c	-	Cone penetration resistance
N	-	SPT number
N_q	-	Bearing capacity factor
β	-	Angle of inclination of principal stress

ABSTRACT

Mechanical behaviour of a granular material is controlled by its constituent particle characteristics. The particle size, gradation, shape and angularity together with the relative density play an important role in the evaluation of deformation characteristics of sands.

In the present study, the effect of particle size on volume change potential, compressibility and shear strength of the fractionated Kalpi sand has been investigated. For test purposes, Kalpi sand has been fractionated into four uniformly graded fractions.

It is observed that e_{max} decreases with particle size, but the effect of D_{50} on e_{min} is insignificant. The uniformly graded fine fractions have been shown to result in a highly loose sand packing compared to coarser fractions. The power law equation proposed by Bellotti et al (1985) and modified by Rahim (1989) is used for formulating the modulus-confining stress relationship for fractionated sands. The effective particle size is shown to have significant influence on the compressibility of sands. Past consolidation stress has been shown to be a significant factor in the study of effect of overconsolidation on the modulus of sands.

It is brought out that the shear strength, initial shear stiffness and dilatancy increases with average particle size. The power law equation proposed by Phool Chand (1987) has been

used for predicting the shear strength. The experimental and predicted values of the equivalent angle of internal friction were found to be in good agreement.

The predicted values of q_c and N have been compared with the available data, and it is observed that the particle size has significant effect on q_c and N values for sands.

CHAPTER 1

INTRODUCTION

In confined compression, the volume of soil decreases as the stress increases. This volume decrease is due to 4 factors namely, the interparticle sliding, the bending of the plate like particles, the deformation of contacts and the crushing of the edges. Compressibility of sand will be more for loose sands compared to dense sands. Relative density of sands alone is not sufficient to characterize the performance of different granular soils. The compressibility of sands depends on a variety of material related and stress level related factors (Clayton et al, (1985), Phool Chand (1987) and Rahim (1989)).

Yudhbir and Rahim (1987) have reviewed factors governing compressibility. According to them the compressibility is high when the sand is loose, the grains are angular, the coefficient of uniformity, Cu, is low, the average particle size is small,particle surface is smooth, the grain mineral strength is small and interparticle cementation is weak. Stress history, stress level and stress path have significant influence on compressibility of sands.

The factors governing the shear strength of a dry granular soil fall into two general groups. The first group includes those factors that affect the shear strength of a given soil viz., the void ratio of the soil and the confining stress. The second group includes those factors that cause the strength of

one soil differ from the other even for the same confining stress and void ratio. The factors included in this group are the particle size, angularity and gradation. In general the factors that govern the shear strength are:

1. Relative density
2. Initial void ratio
3. Confining stress
4. Stress history
5. Degree of interlocking
6. Particle angularity
7. Average particle size
8. Mineral type
9. Particle roughness
10. Gradation
11. Stress path
12. Dilatancy
13. Mineral to mineral friction angle.

While effect of some of these factors on the mechanical behaviour of sand has been investigated, very little systematic research work has been carried out in respect of effect of particle size. In this study, it was decided to investigate the stress-strain-volume change behaviour for 4 different relative densities under normally consolidated dry state for 4 different size fractions of Kalpi sand. Dependence of modulus and shear strength on particle size, confining stress and relative density have been investigated.

CHAPTER 2

LITERATURE REVIEW

In the following brief literature review an effort has been made to examine the factors governing the volume decrease potential, (e_{max} - e_{min}), compressibility and strength of sands. Behaviour of sands is predominantly governed by the material related and stress level related factors such as mineralogy, grain morphology, in-situ density, stress history, stress level, angularity, gradation and particle size. In natural sands most of these factors are interrelated and it is impossible to differentiate the influence of one factor from the other.

One of the earliest investigations concerning the compressibility of sand was reported by Terzaghi (1925). He performed a number of confined compression tests on sand and on clayey soils and observed that the compressibility of both types of soils was similar. However, the volume changes in the sand were not as great as those which occurred in clayey soil when tested under the same magnitude of load. Nevertheless, the sand was found to be quite compressible, even at pressures of the order of 3 kg/cm^2. An extension of this early work is quoted by Terzaghi and Peck (1948). A number of sands and sand mica mixtures were subjected to one dimensional compression. Particle crushing was observed and was suggested as a cause for the compressibility of the sand at high pressures.

Other one dimensional compression tests on sand have been reported by Casagrande (1936), Rowe (1955), Jakobson (1957),

Roberts and De Souza (1958) and Schultz and Moussa (1961). The results of each of these investigations are qualitatively the same as the early work reported by Terzaghi.

Jakobson (1957) conducted tests on two visually identical standard sands, but one of them having more polished grains and reported that there was a marked difference in the $(e_{max}-e_{min})$ of the two sands. The ϕ' value of the sand having polished grains was less than that of the other one.

Roberts and De Souza (1958) tested different sands and observed that at moderately low pressures angular sands crushed and compressed more than rounded sands. However, at very high pressures little difference was observed between the behaviour of angular sand and rounded sand. They also observed that a uniformly graded soil crushes more than a well graded soil.

Schultz and Moussa (1961) observed that compressibility of a sand is considerably increased due to mixing it with a small quantity of cohesive soils. Also the moist sand is less compredssible than dry sand while saturated sand behaves, as gathered from other tests, like dry sand.

Kolbuszewski and Fredrick (1963) observed slight increase in compressibility as roughness of pa~~s~~rticles increases for dense sample and a large increase in compressibility as roundness of particles decreases for loose samples. They also reported that ϕ increases as the particle size increases.

Kjaernsli and Sande (1963) reported that for a given material, compressibility is low for a well graded soil in comparison to uniformly graded material.

Vesic (1963) conducted triaxial test on medium sand from Chattahoochee river, which is rich in mica and made of subangular quartz particles. He plotted friction angle (Φ') versus void ratio (e) and approximated it to the equation

$$\tan \Phi' = \frac{0.59}{e}$$

Kirkpatrick (1965) has done tests on glass and Leighton Buzzard sand within the range of medium to coarse sand sizes (2 mm to 0.3 mm), and in each material the particle shape and surface texture were uniform throughout this range. He reported that ϕ' increases as the grain size decreases. He also mentioned that the conclusions made by him about the influence of particle size on the shearing resistance are contradicted to the results reported by Kolbuszewski and Frederick (1963). Also as observed by the author, the position of ϕ' - relative porosity lines is sensitive to the value of limiting porosities when compared at equivalent porosities. Dilatancy decreases as the porosity increases for equivalent sizes.

Bishop (1966) reported that the curvature of the failure envelope is most marked for soil which

(a) are initially dense or heavily compacted

(b) are initially of relatively uniform grain size

(c) if undisturbed, have been heavily over-consolidated.

Lee and Seed (1967) obtained final void ratio less than the minimum void ratio obtained from laboratory procedures for an initially loose specimen of fine uniform sand from Sacramento river at confining pressures greater than 100 kg/cm^2.

Lee and Farhoomand (1967) observed from the isotropic and anisotropic triaxial compression tests that,

(a) granular soil is quite compressible under an applied load

(b) compression is usually accompaniedby a certain amount of particle breakage, and the two phenomena seem to be somewhat related to each other.

(c) Coarse soils compress more and show more particle crushing than fine soils under large stress

(d) Soil with angular particles compress more and show more particle crushing than soil with rounded particles

(e) Uniform soils compress and crush more than graded soils with the same maximum grain size

(f) Under any particular load, compression and particle crushing continue to increase at an ever decreasing rate for an indefinite period of time.

Vesic and Clough (1968) reported that below 1 kg/cm^2, there is very little crushing, the sand particles are relatively free to move with respect to each other and the dilatancy effects gradually disappear. Crushing appeared to be intense in the elevated pressure range (10 to 100 kg/cm^2), until the breakdown stress was reached. Breakdown stress is defined as the stress needed to eliminate all effects of the initial void ratio of sand.

Lambe and Whitman (1969) concluded that crushing and fracture of particles actually begins in a minor way at very small stress, but becomes increasingly important when some critical stress is reached. This critical stress is smallest when the particle size is large, the soil is loose, the particles

are angular, the strength of individual mineral particle is low.

EL-Sohby (1969) pointed out that the total deformation of a mass of sand under any stress system can be divided into two main components: (a) Elastic deformation, due to the sum of the elastic deformation of the individual particles, and (b) Sliding deformation, due to the sliding of particles relative to each other.

Koerner (1970) conducted tests on quartz, feldspar and calcite with different particle shape, gradation and particle size, evaluated by varying effective size, D_{10}. He observed that the angle of internal friction at large strain is more when the angularity is more, Cu is more and the effective size is less at saturated condition. His results also suggest that dilatancy is independent of particle angularity. Perhaps this conclusion, which is influenced by the membrane penetration consideration is responsible for his other conclusions regarding the variation of residual angle of shearing resistance.

EL-Sohby and Andrawes (1972) reported that the compressibility of a dense packing is nearer to that of an ideal elastic isotropic material than that of a loose packing. The compressibility increases with increase in porosity and particle angularity. Uniformly graded sands are generally more compressible than graded sands. The compressibility also depends on the elastic properties of the individual particles. They also observed that particle size alone has no significant effect on the compressibility, from the tests conducted on 3 Glass

Ballotini samples having uniform particle sizes 0.1,0.2 & 0.3 mm.

D'Appolonia and Holubec (1973) observed that the particle shape has minor influence on the minimum void ratio, but has a major influence on the maximum void ratio. Since the maximum void ratio increases considerably more with angularity than does the minimum void ratio, the (e_{max} - e_{min}) diverges markedly with increase in angularity. They observed that the deformability of sands having the same gradation and relative density increased with increasing angularity. Also, strength increased with increasing angularity.

Stamatopoulos and Kotzias (1973) concluded the importance of constrained modulus over compression index as:

1) The constrained modulus is analogous to the measure of deformation used in other materials such as concrete and steel.

2) The constrained modulus provides a linear rather than a logarithmic function and is easier to apply to calculations leading to predictions of settlements.

3) The overconsolidated soils the concept of compression index cannot be used as such for predicting settlements.

Tassios and Sotiropoulos (1973) observed that the critical strains are increased with smaller loaded masses, larger grain sizes, higher confining pressures and gradual breakage to repetitive loadings.

Ishihara and Watanabe (1976) conducted tests on commercially available glass beads and Fuji river sand of same D_{50} values and observed that (e_{max} - e_{min}) decreases when the average grain size increases. They also pointed out that when the angularity increases the (e_{max} - e_{min}) also increases. Youd (1973) observed

that the particle size has very little effect on e_{max} and e_{min}.

Grivas and Harr (1980) conducted tests to determine types of movements that would take place during the application of load. They observed that sliding predominates at the upper and lower surfaces of the specimen and in these regions rotation is at a minimum. But rotation is far more likely to occur at the curved surface of the specimen than within its interior.

Juarez-Badillo (1981) observed that for granular materials, in the unloading compression curves, the coefficient of compressibility increases with size, relative density, cycle number and cycle amplitude in cyclic loading. The swelling coefficient appeared to be independent of size, relative density and number of cycles.

Janbu (1985) proposed an equation for finding out the modulus during unloading,

$$M = M_{sw} \cdot \sigma'$$

where

$$M = \frac{d\sigma'}{de}$$

M_{sw} = slope of the swelling curve

σ' = effective vertical stress.

Bellotti et al (1985) related the constrained modulus M by normal stress and relative density as

$$M = m_o P_o \left(\frac{\sigma'}{P_o}\right)^{m_1} \times \exp(m_2 \times I_D)$$

where,

P_o = reference stress (say 1 kg/cm^2),

m_o, m_1, m_2 = experimental constants.

Rahim (1989) pointed out that these experimental constants are governed by mineralogy and angularity characteristics of sands.

Skempton(1986) reviewed the available data for different sands with a relative density range of 40-80% and effective overburden pressure less than 2 ton/ft^2 and observed that at a given relative density and overburden pressure, the blow count N is higher for sands with a larger mean grain size (D_{50}).

Yudhbir and Rahim (1987) have reviewed the factors controlling compressibility. They observed that in general the compressibility is high when the sand is loose, the grains are angular, the coefficient of uniformity is low, the average particle size is small, particle surface is smooth, the grain mineral strength is small, interparticle cementation is weak and the ageing effects are insignificant. Stress history, stress level and stress path have significant influence on compressibility of sands. They conducted tests on sands of different mineralogy, gradation and particle shape and observed that the angular well graded sands attain closer packing of particles compared to uniformly graded and rounded sands for same height of fall. Also angular well graded sands are more compressible than uniformly graded rounded sands. They also pointed out that in case of angular sands, the grain boundaries

are modified at a stress level less than the critical stress, and at the stress level greater than the critical stress, sands behaves like clays and exhibits high compressibility.

Phool Chand (1987) investigated the response of three sands, namely standard rounded quartz sand, angular micaceous Ganga sand and angular Kalpi sand under one dimensional compression and direct shear tests. The modulus during unloading was shown to follow the relationship proposed by Janbu (1985). The rounded quartz standard sand shows less compressibility and the micaceous angular Ganga sand shows high compressibility. The shear strength of all the sands was observed to be almost equal. He also mentioned that at a given confining pressure the modulus (M) and shearing stress increase with relative density at a much faster rate in case of incompressible rounded quartz sand and the corresponding changes are less marked in case of compressible angular sands.

Rahim (1989) observed that grain size, shape, angularity, mineralogy, gradation and relative density are the factors governing the degradation of natural sand under stress. The compressibility of sands is strongly controlled among other factors by mineralogy and grain angularity. At low stresses, the stress-strain behaviour of weak sands is mainly governed by grain interlocking. At higher stress (> 30 kg/cm^2), the stress-strain behaviour is significantly different from that at low stresses due to the grain modification and crushing. The power law proposed by Bellotti et al (1985) and the suggested coefficients m_o, m_1, and m_2 are strictly applicable for stress levels less than critical stress.

This brief review of literature would suggest that while effects of particle angularity, mineralogy and relative density on volume change potential, compressibility and shearing resistance of sands have been examined in some detail, no systematic investigation into the influence of grain size on the engineering behaviour of sands is available. The present investigation attempts to investigate the effect of grain size on:

1. The behaviour of uniformly graded sands with varying D_{50} values on $(e_{max} - e_{min})$.
2. The response of four sands with different D_{50} values but same mineralogical composition during one dimensional compression.
3. The stress-strain-volume change behaviour of four sands with same mineralogy,comparable grading and angularity but different particle size under different normal stresses during direct shear tests.

It was decided to examine the suitability of some of the available mathematical models to determine the shear strength and constrained modulus at different confining stress and relative density.

CHAPTER 3

EXPERIMENTAL INVESTIGATIONS AND DATA PRESENTATION

3.1 General

In the present day Geotechnical Engineering the behaviour of soils is studied either in-situ or in the laboratory. Although, the in-situ tests are uneconomical and laborious, they give~~s~~ more or less accurate results. But to compromise with the economical aspect, engineers try to go in for laboratory tests, which generally give reasonably accurate values of the soil parameters. Since it is difficult to obtain undisturbed samples, especially in the case of sands, remoulded samples can be prepared to perform the experiments in the laboratory.

3.2 Material Used

For this study, the sand obtained from Jamuna river in northern part of India, popularly known as Kalpi sand has been used. Kalpi sand is a well graded, light brown soil with angular grains having mineralogical contents of quartz, feldspar, carbonate and very small amount of mica. For test purpose, the well graded sand was fractionated into four sizes as shown in table 3.1.

Table 3.1

Sl. No.	Designation	Description
1	0.212 mm	Passing through 0.425 mm and retained on 0.212 mm sieve
2	0.425 mm	Passing through 0.60 mm and retained on 0.425 mm sieve
3	0.85 mm	Passing through 1.0 mm and retained on 0.85 mm sieve
4	1.70 mm	Passing through 2.0 mm and retained on 1.7 mm sieve

The above mentioned designation has been used to represent the sands unless otherwise stated.

Kalpi sand was treated with hydrochloric acid and the effervescence showed the presence of calcium carbonate coating over the grains. It implied that the quartz particles had travelled through a carbonate rich area and hence the coating.

3.3 Experiments Performed (See Table 3.2.1, Experimental Programme)

3.3.1 Grain Size Distribution

The general Kalpi sand was analysed by a set of sieves (4.75 mm, 2.0 mm, 1.70 mm, 1.0 mm, 0.85 mm, 0.60 mm, 0.425 mm, 0.212 mm and 0.15 mm). The sieve set with sand was vibrated for 15 minutes in the mechanical shaker. The D_{50} of the sand has been found out as 1.06 mm and the Cu as 4.34. Grain size distribution is shown in fig. 3.1.

3.3.2 Maximum Void Ratio

The maximum void ratio was determined by pouring the sand into a calibrated container with the help of a glass funnel by keeping the height of fall practically zero. The maximum void ratio was taken as the average of 5 readings which gave only a second decimal variation. Results are given in Table 3.2

3.3.3 Minimum Void Ratio

The minimum void ratio was found out by adding sands in 3 layers into a standard mould (volume = 3000 cm^3) and each layer was vibrated for 15 minutes under the surcharge of 0.14 kg/cm^2. The average of 3 readings which gave only a second decimal variation was taken as minimum void ratio (See Table 3.2).

3.3.4 Relative Density Vs. Height of Fall Relationship

Relative density of sand can be find out by raining the sand from a hopper which is positioned at a fixed height (raining technique). The relative density of sand is a function of height of free fall as long as the rate of flow of sand remains constant. The relative density of sand was determined by varying the height of fall, but keeping the hopper opening constant (Fig.3.2(a). The raining technique was used to obtain reproducible sand samples.

3.3.5 Sample Preparation

Samples were prepared by free fall of sand from the hopper for 4 different relative densities, viz. 20%,40%, 60% and

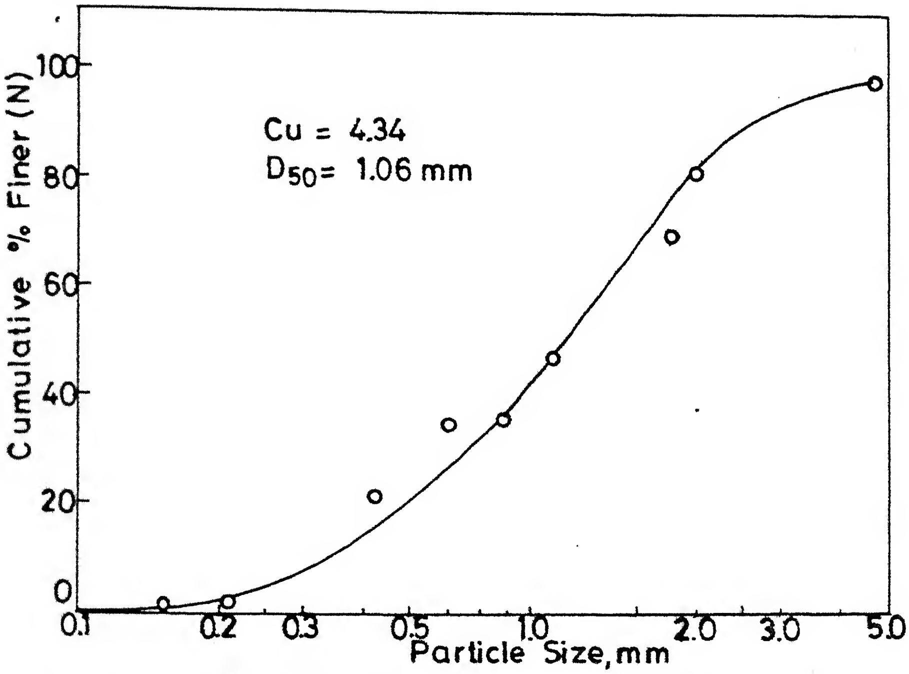

Fig. 3.1 Grain Size Distribution

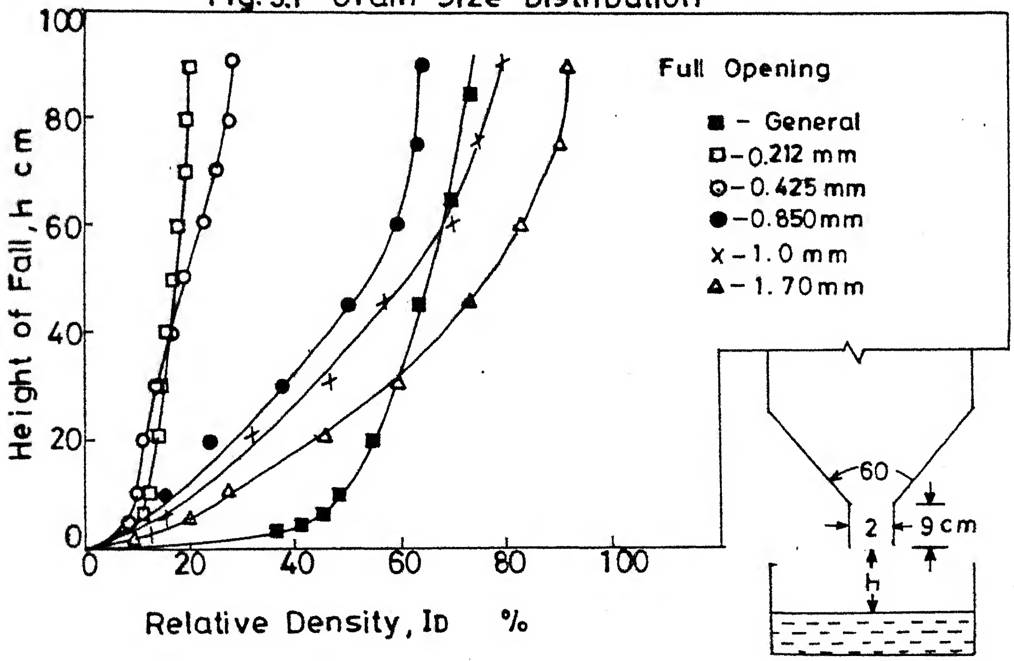

Fig 3.2(a) Relationship Between Height of Fall and ID.

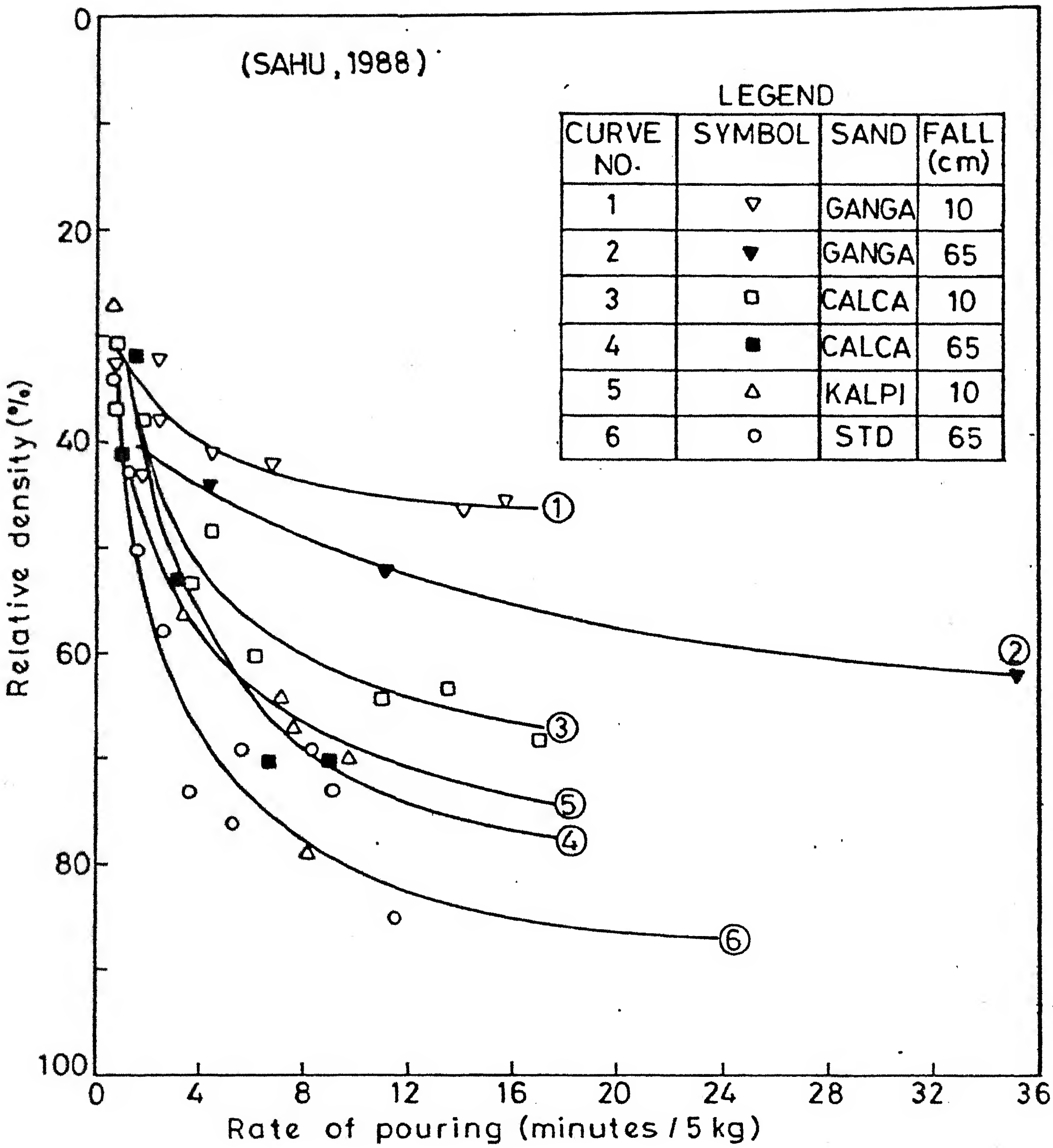

Fig.3.2(b) Variation of relative density with rate of pouring for all sands

Table 3.2

Physical Properties

Size, mm	G	D_{10}	D_{50}	D_{60}	Cu	e_{max}	e_{min}	$(e_{max}-e_{min})$
0.212	2.66	0.23	0.32	0.34	1.48	1.08	0.69	0.39
0.425	2.66	0.44	0.51	0.53	1.20	1.03	0.64	0.39
0.850	2.66	0.87	0.92	0.94	1.08	0.97	0.64	0.33
1.0	2.66	1.07	1.35	1.42	1.33	0.92	0.61	0.31
1.70	2.66	1.73	1.85	1.88	1.09	0.89	0.62	0.27
General	2.66	0.30	1.06	1.32	4.34	0.83	0.48	0.35

80% for all the 4 types of sands. Rate of fall is an important factor in controlling the relative density of samples (Sahu, 1988). Rounded sand attained more relative density than angular sand at equivalent rate of fall (Fig. 3.2(b)). The curve for Kalpi sand is relevant to the present study. The height of fall versus relative density relationship has been used as a guide line for sample preparation. The hopper opening and height of fall have been varied by trial and error to get the desired relative densities. Maximum observed scatter in the value of relative density was less than 5%.

3.3.6 Oedometer Compression Test

The oedometer test was performed in the laboratory to get the informations about the compressibility characteristics of different size fractions of the Kalpi sand in the dry state.

Cyclic loading for all the sands were performed for four different relative densities at stresses 0.25, 0.5, 1.0, 2.0, 4.0, 8.0 and 16.0 kg/cm^2 and unloaded after each increment of stress. The results are plotted in terms of void ratio (e) or compression versus the logarithm of stresses (logσ') in Fig. 3.3(a) to Fig.3.3(d).

3.3.7 Direct Shear Test

The box shear apparatus was used to test the shear strength characteristics of the sands. Two dial guages were provided for measuring the horizontal movement of shear box and compression/expansion of the sample. The strain rate used was

0.25 mm/minute.

Direct shear test was conducted at normal stresses of 0.50, 1.0, 2.0 and 4.0 kg/cm^2 for 4 sands for relative densities 20%, 40%,60% and 80% in normally consolidated state. Dry sand was used to perform the tests. The results were plotted in terms of shear stress versus displacement in Fig. 3.4(a) to (d) and Fig. 3.5(a) to (d).

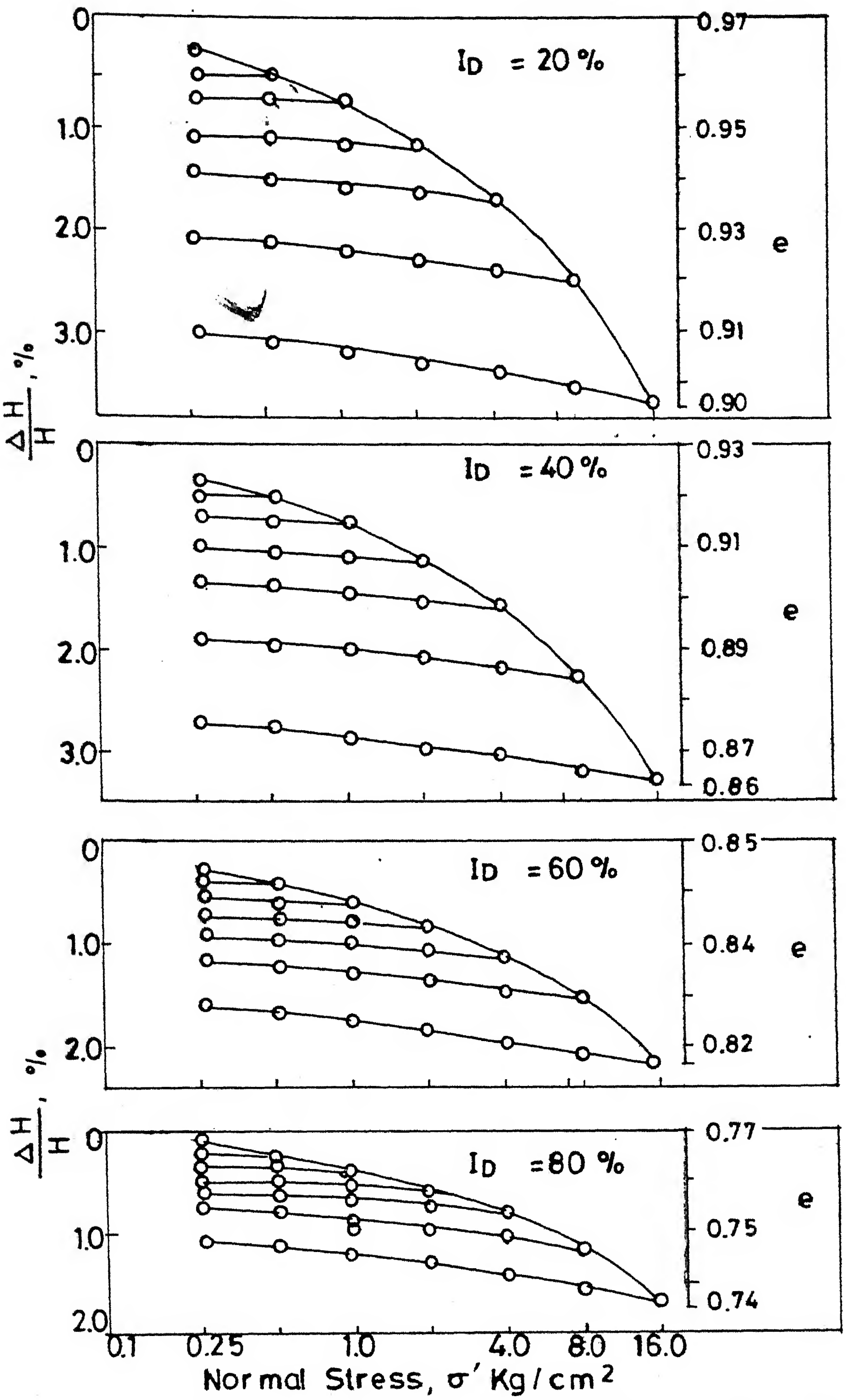

Fig 3.3(a) elog σ' Curve for 0.212 mm Size Sand.

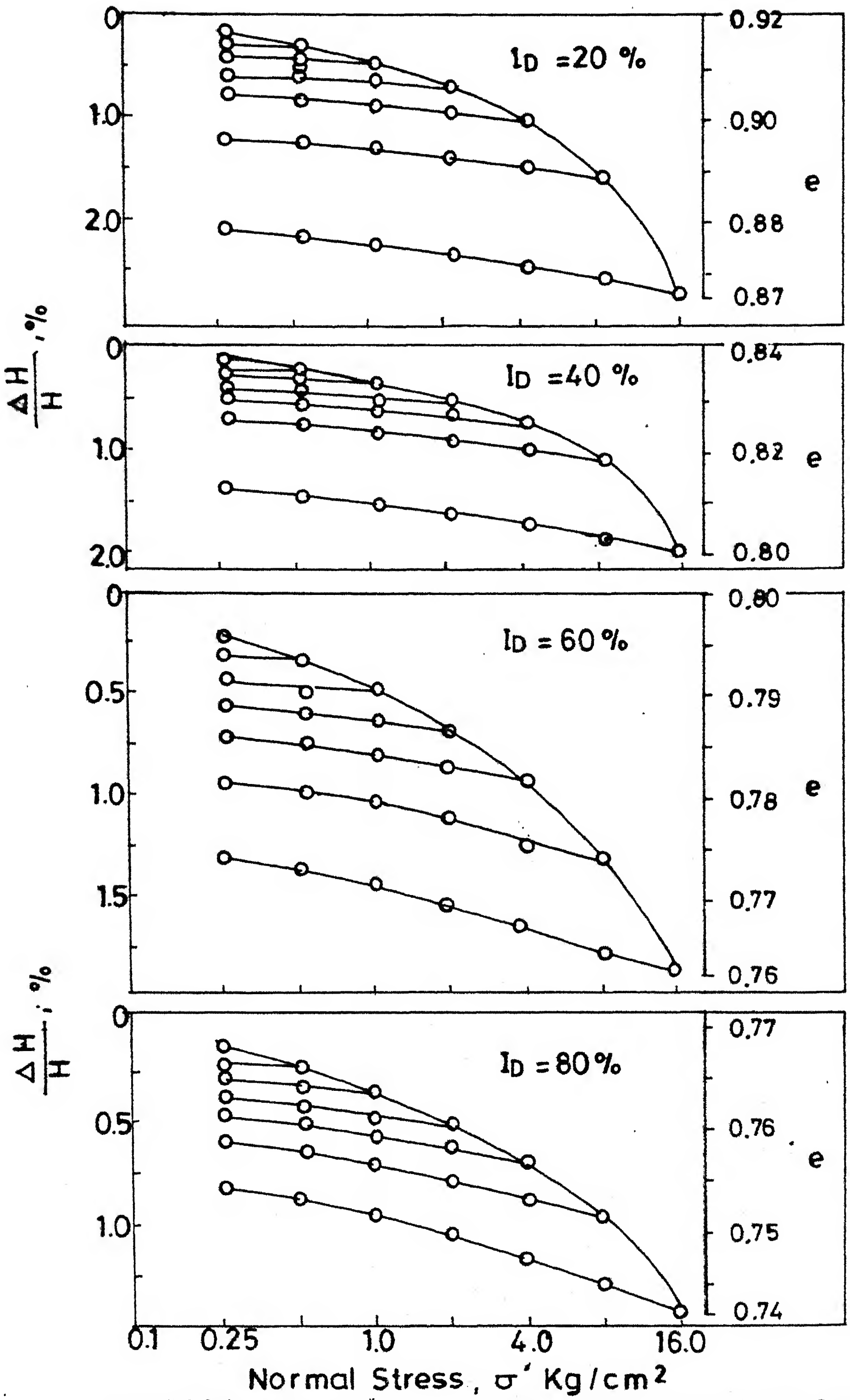

Fig 3.3 (b) e log σ' Curve for 0.425 mm Size Sand.

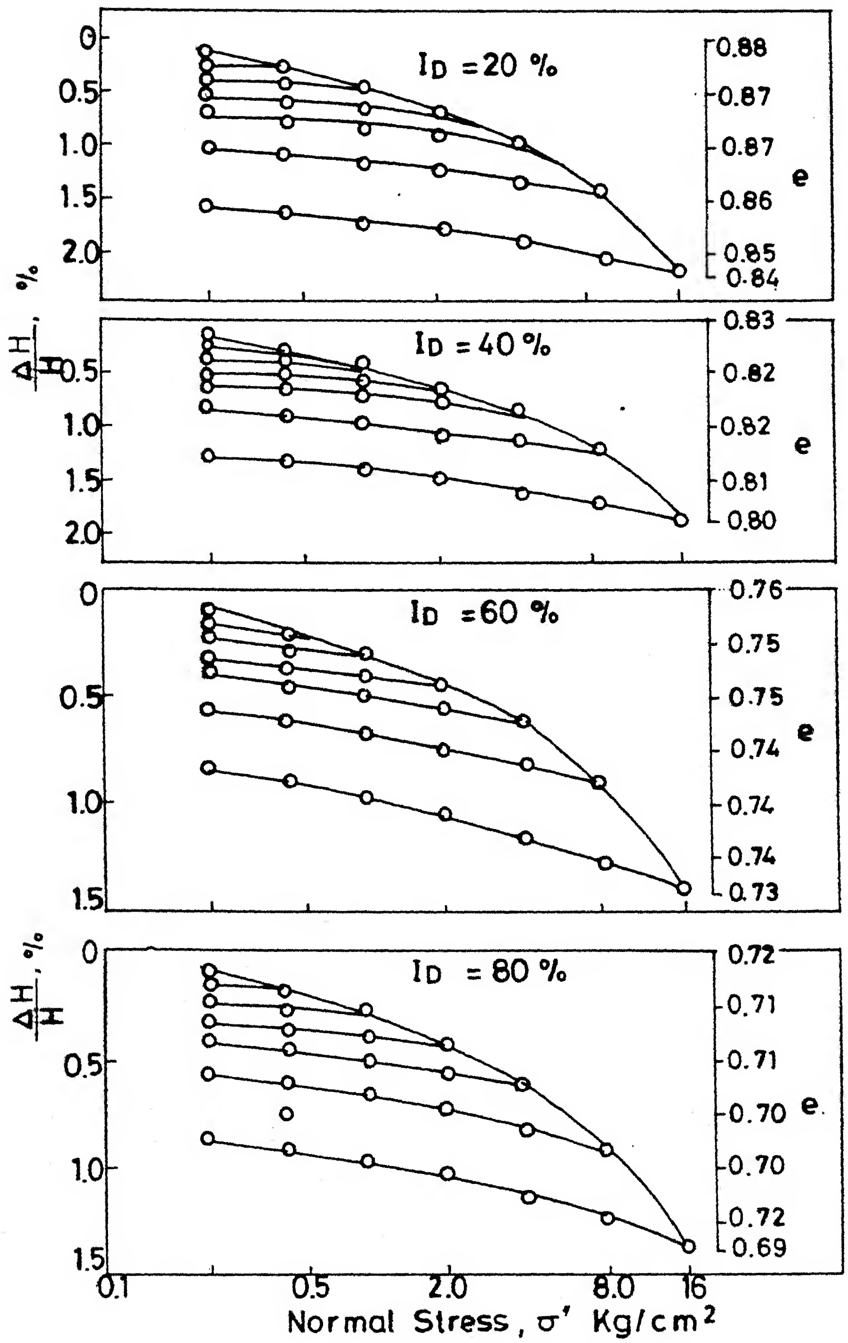

Fig.3.3(c) elog σ' Curve for 0.850 mm Size Sand.

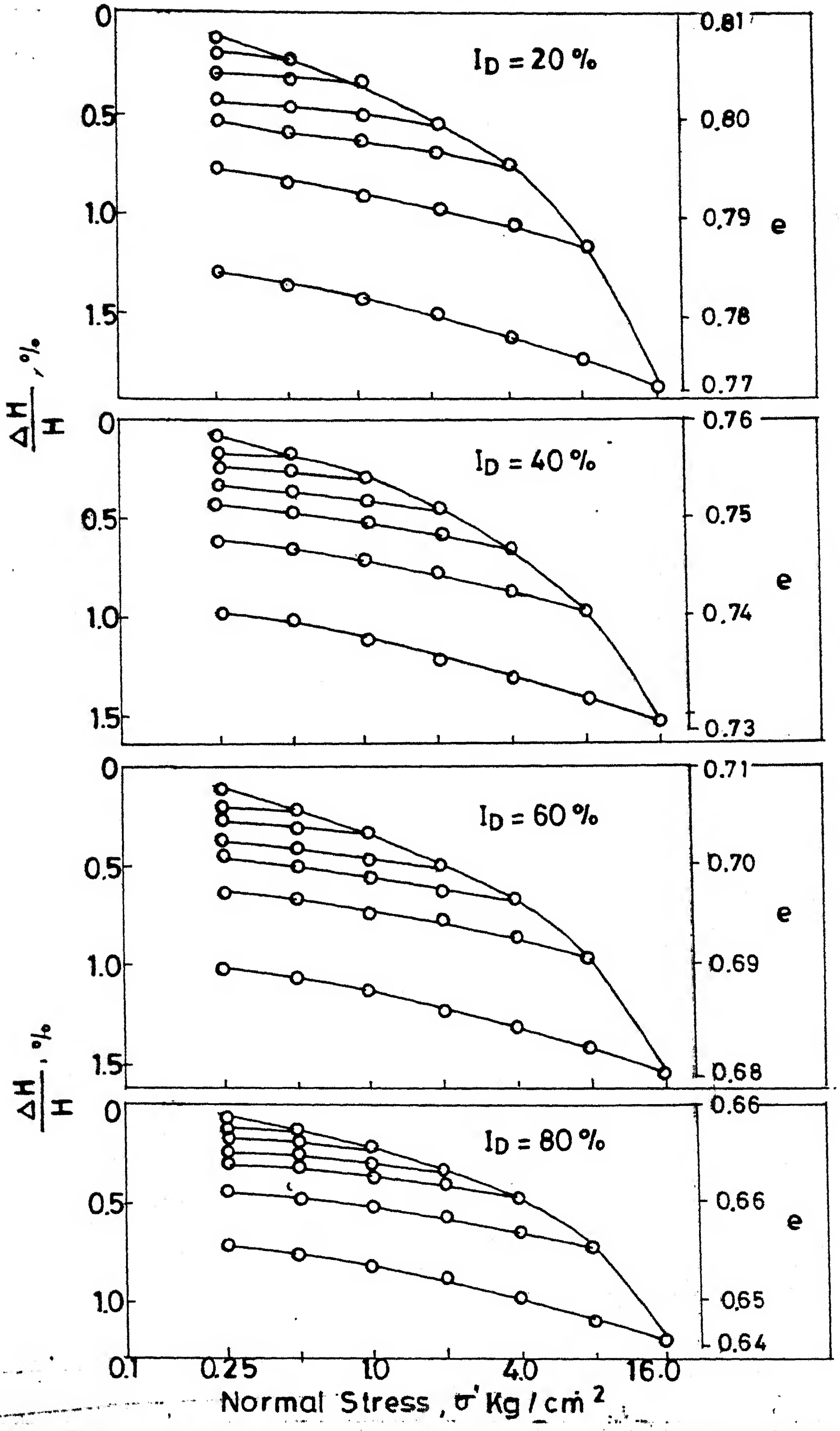

Fig. 3.3 (d) e-log σ' Curve for 1.70 mm Size Sand.

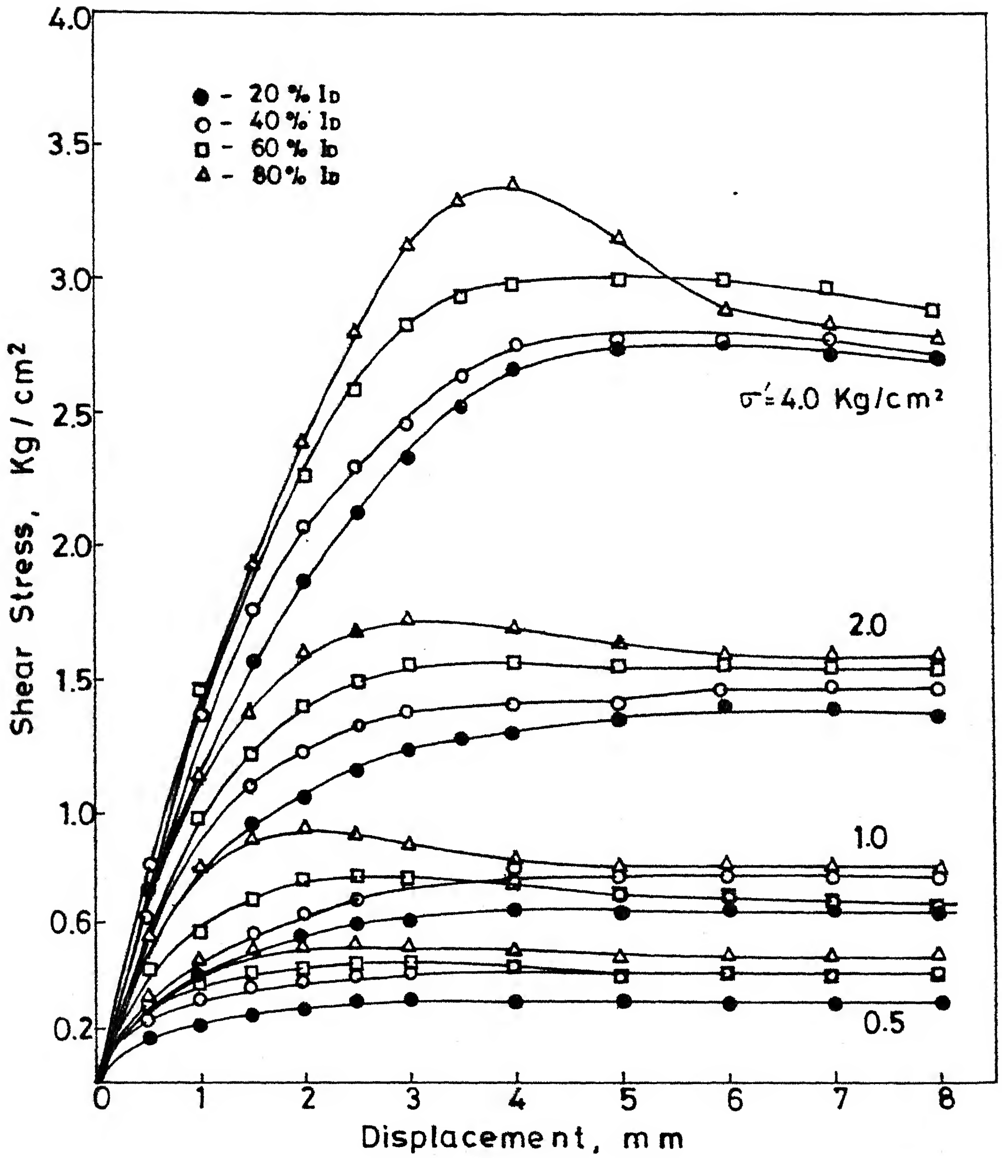

Fig. 3.4(a) Stress Strain Curve for 0.212 mm Size Sand.

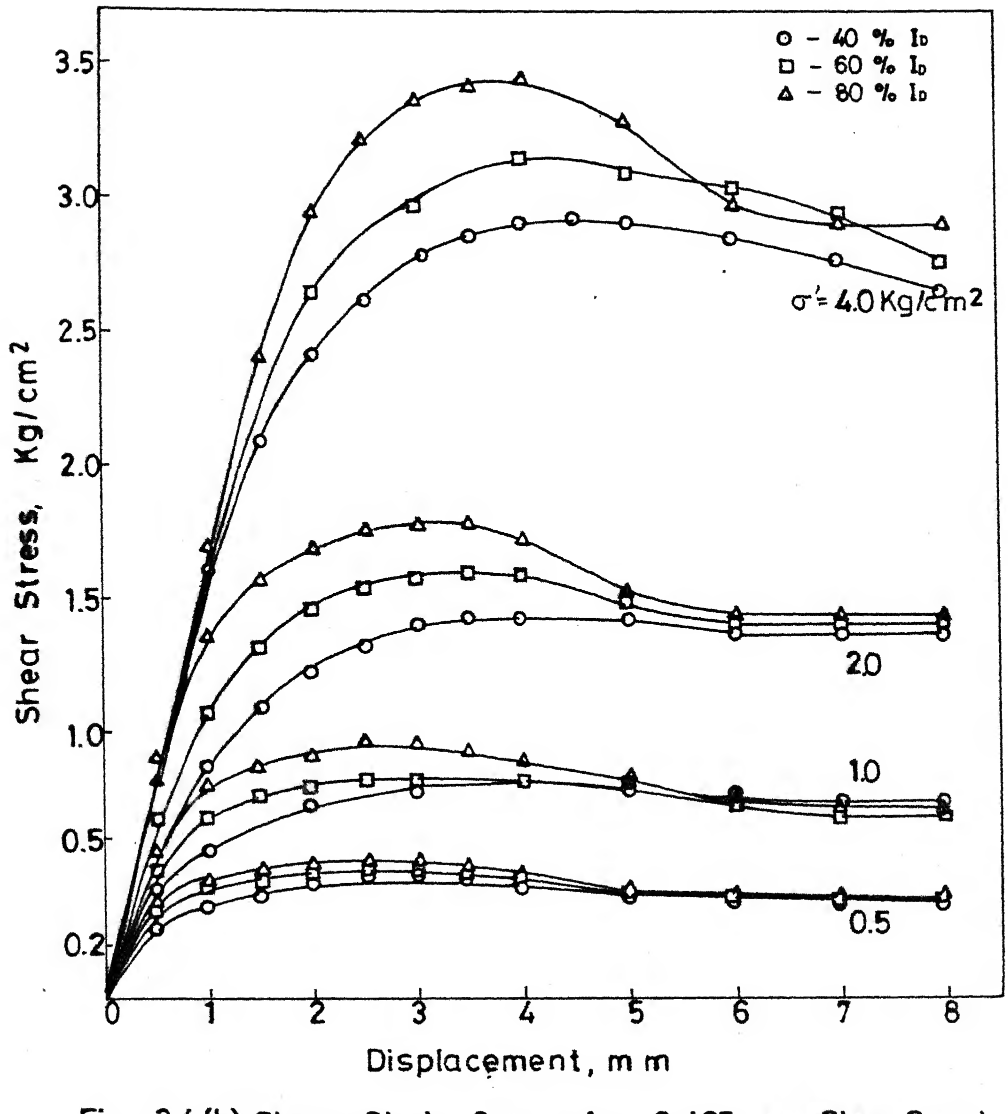

Fig. 3.4 (b) Stress Strain Curve for 0.425 mm Size Sand.

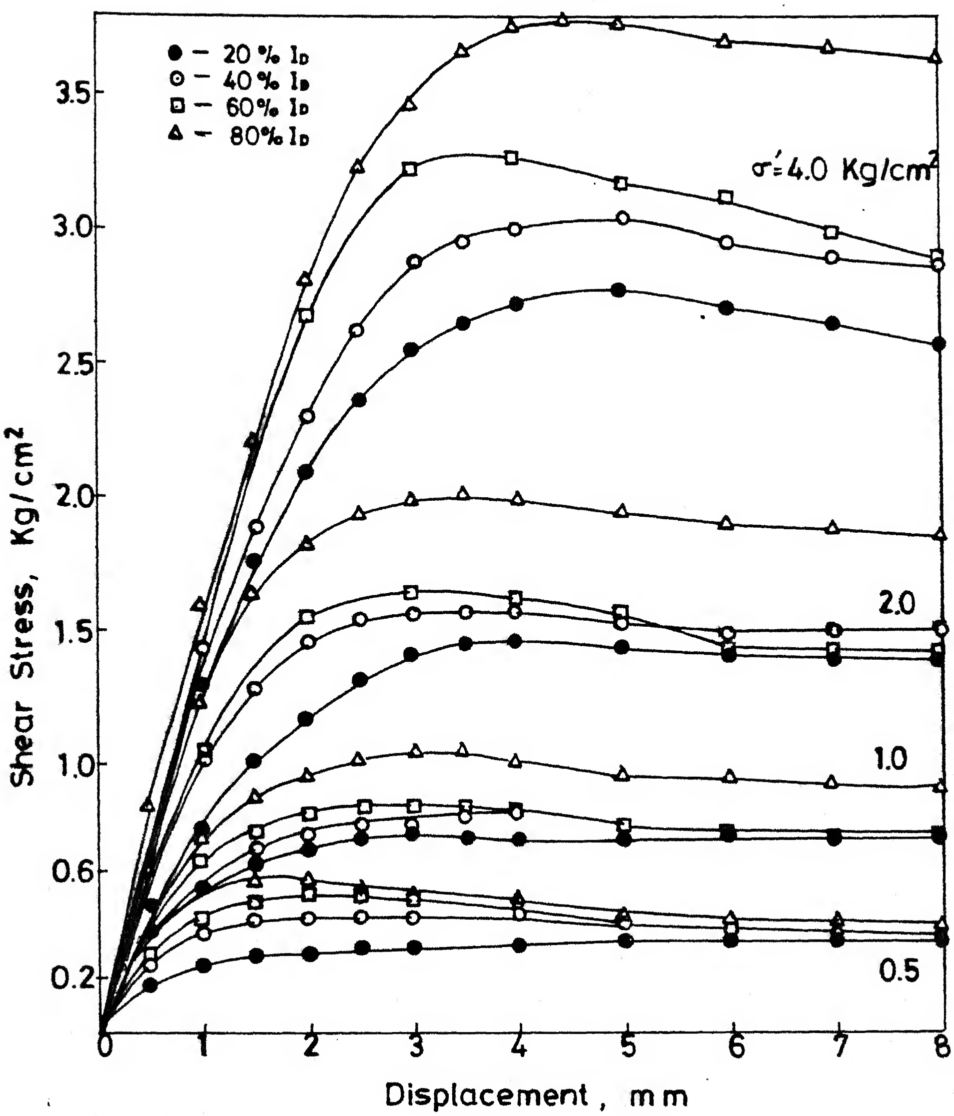

Fig. 3.4 (c) Stress Strain Curve for 0.85 mm Size Sand.

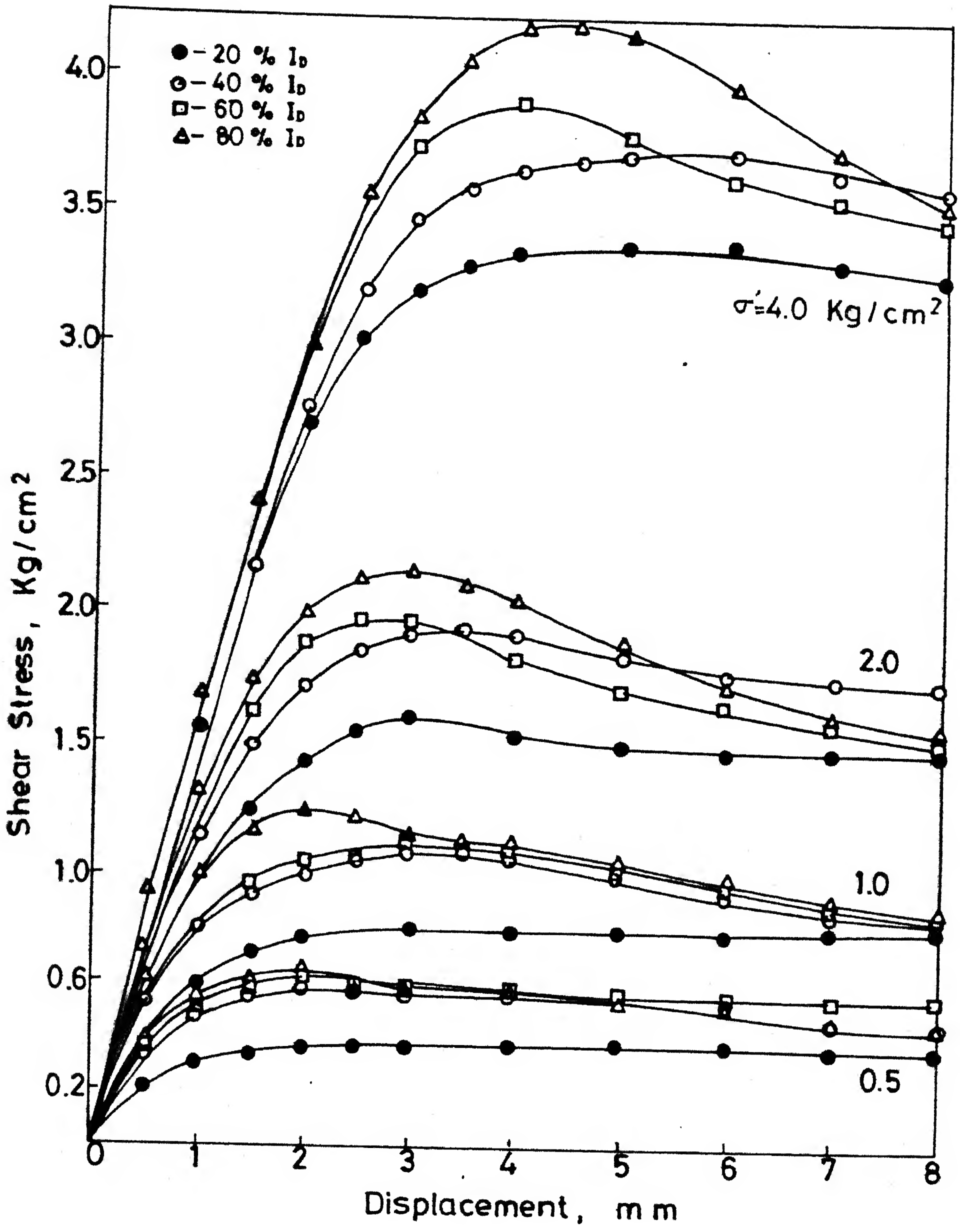

Fig. 3.4(d) Stress Strain Curve for 1.70 mm Size Sand.

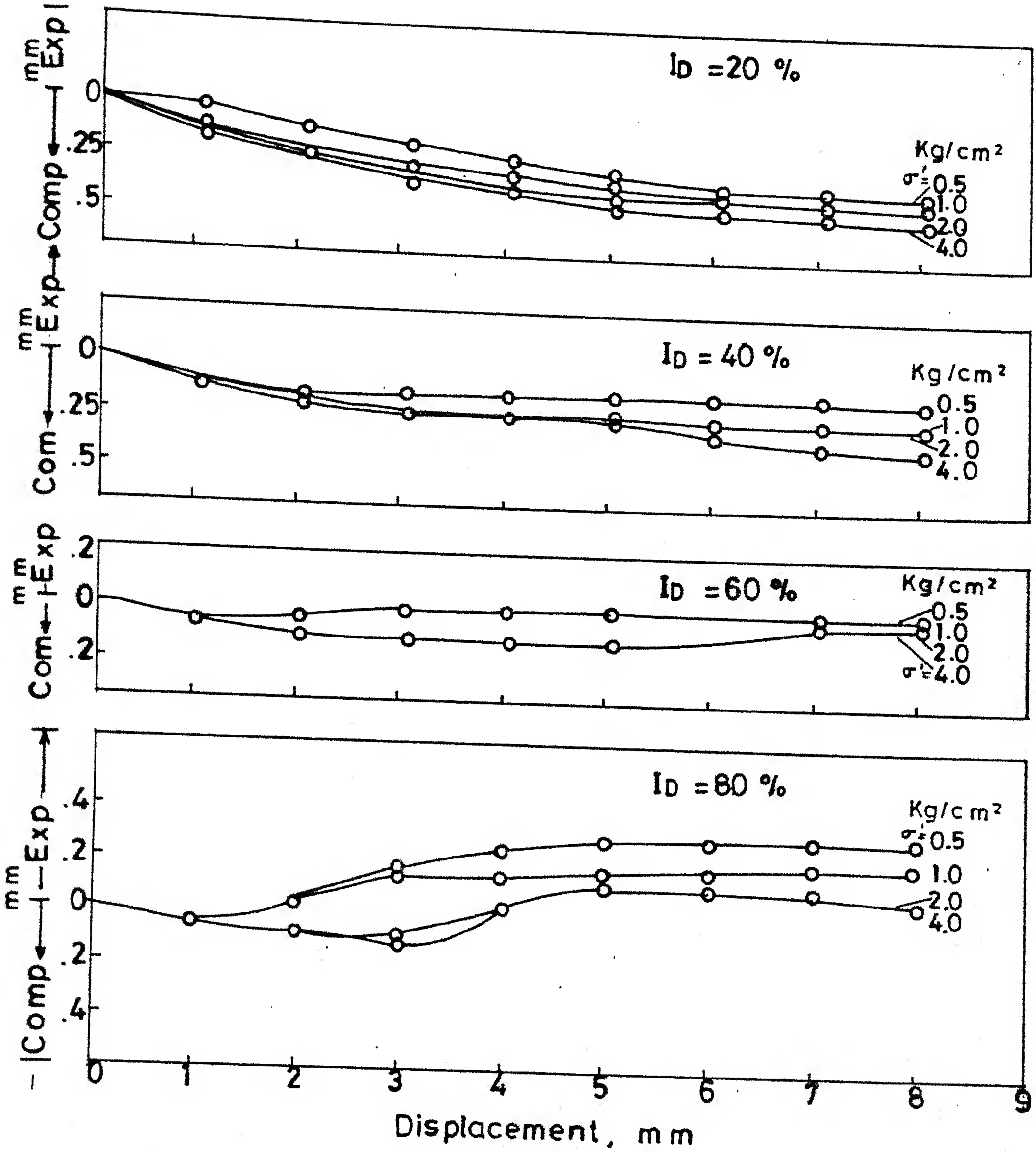

Fig.3.5(a) Compression / Expansion of 0.212 mm Sand.

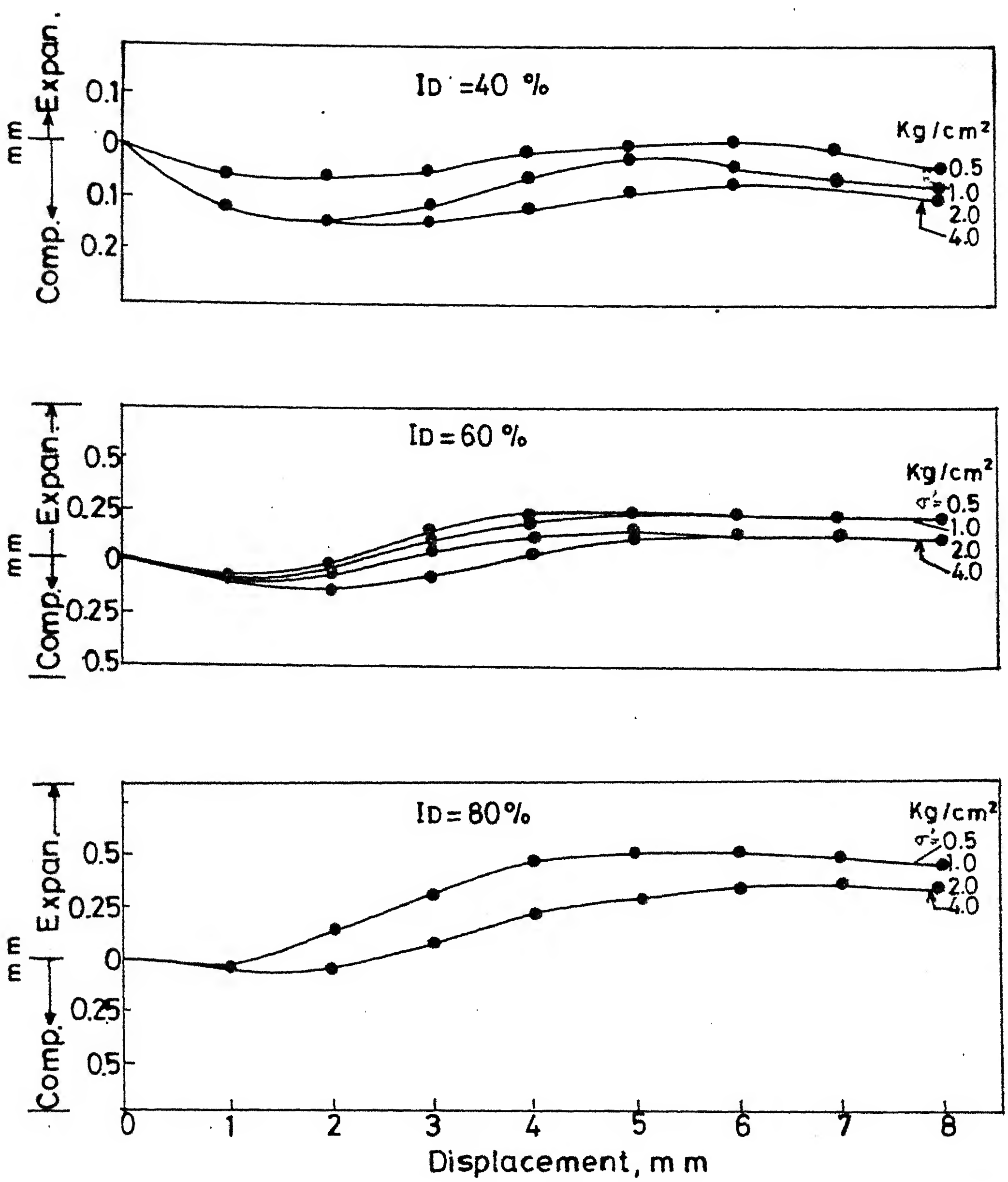

Fig.3.5(b) Compression / Expansion of 0.425 mm Size Sand.

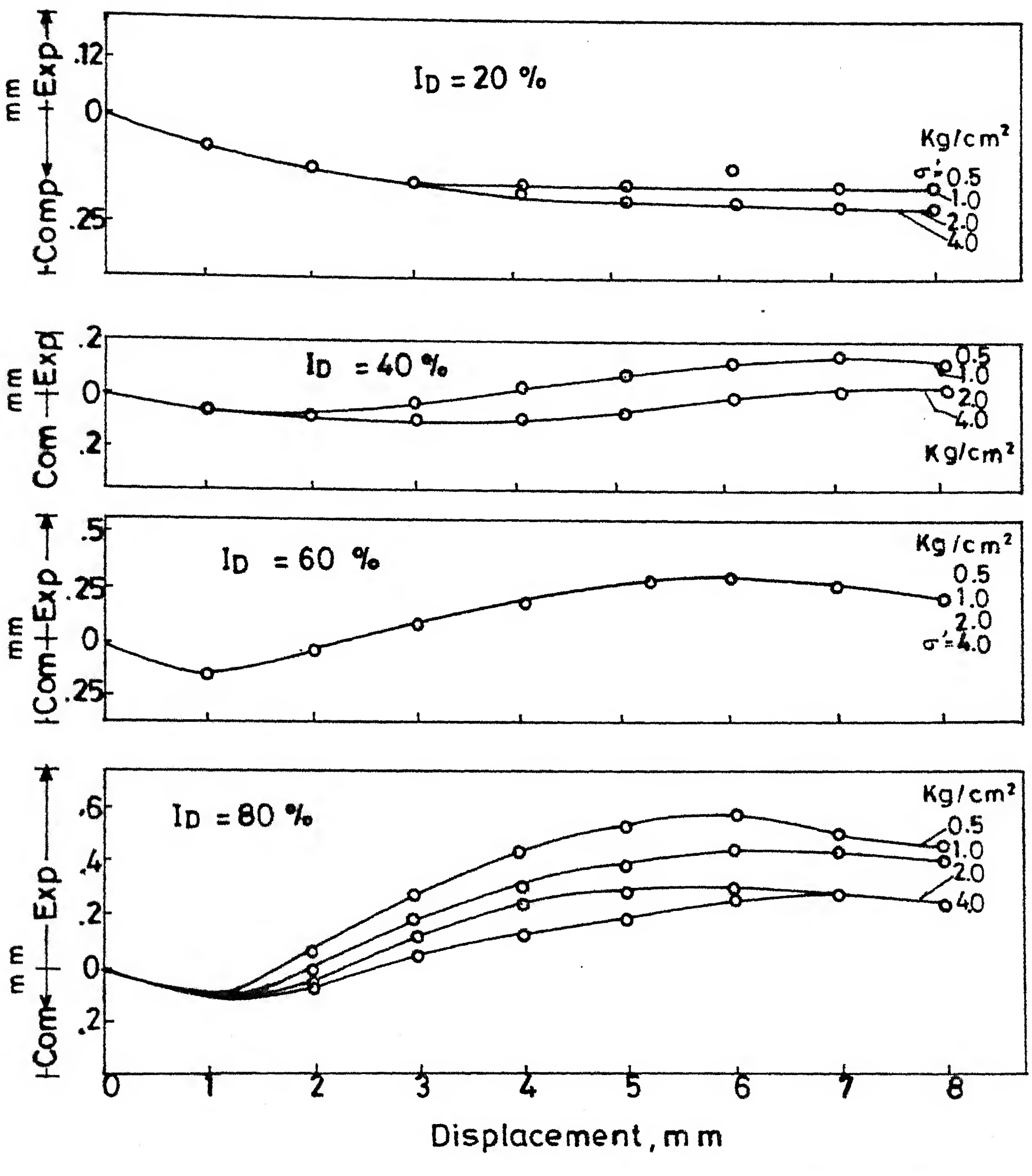

Fig. 3.5 (c) Compression / Expansion of 0.850 mm Size Sand .

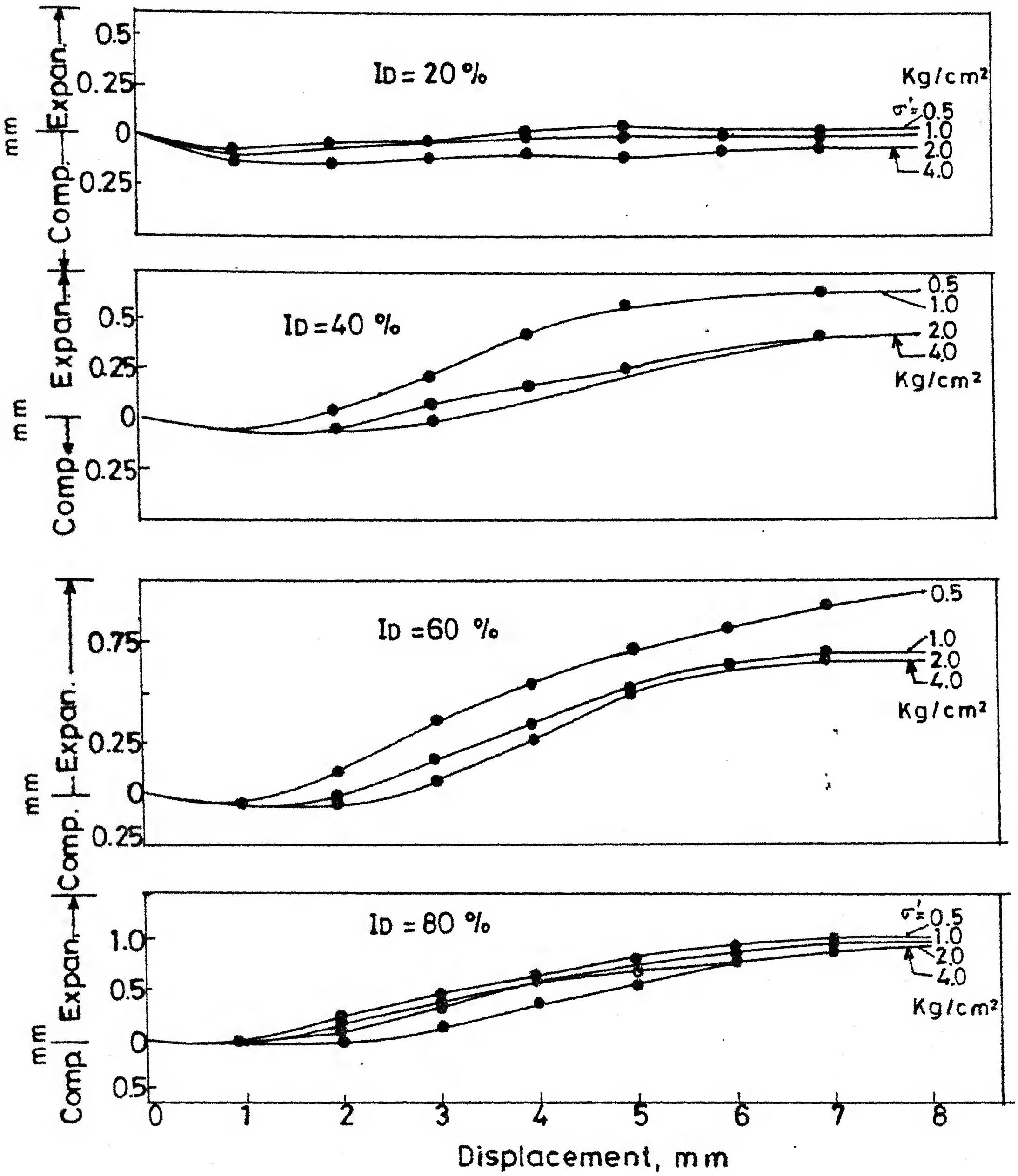

Fig. 3.5 (d) Compression / Expansion of 1.70 mm Size Sand.

Table 3.2.1

Experimental Programme

Sl. No.	Types of Tests Conducted	No. of tests
1.	Sieve analysis	1
2.	e_{max} determination (for 5 fractions and whole sand)	54
3.	e_{min} determination (for 5 fractions and whole sand)	30
4.	Height of fall Vs. relative density relationship (for 5 fractions and the whole sand)	116
5.	Oedometer test: upto 16 kg/cm^2	16
6.	Direct shear test on	
	a. 0.212 mm	16
	b. 0.425 mm	12
	c. 0.85 mm	16
	d. 1.70 mm	16
	for I_D = 20%, 40%, 60%, 80% and each I_D value under = 0.5, 1.0, 2.0 and 4.0 kg/cm^2 except 0.425 mm fraction for which test for I_D = 20% was not performed	

CHAPTER 4

DATA INTERPRETATION AND DISCUSSION

4.1 General

In this chapter the data presented in the previous chapter have been analysed. Also an attempt has been made to correlate the experimental data with some of the available mathematical models for constrained modulus and shear strength.

4.2 Effect of Particle Size on Limiting Void Ratios

On the basis of test results obtained during this study and those reported by Alyanak (1961), Frederick (1961), Youd (1973) and Ishihara and Watanabe (1976), the effect of particle size (chosen as D_{50}) on e_{max}, e_{min} and (e_{max}-e_{min}) is illustrated in figs. 4.1(a) and (b). While e_{max} shows variation with D_{50}, the value of e_{min} is practically constant over a wide range of particle size. The rate of change of e_{max} (Fig. 4.1(a)) and e_{max}-e_{min} (Fig. 4.1(b)) with D_{50} is influenced by the angularity of sand. As already brought out by Youd (1973), angular sands have higher e_{max} and (e_{max}-e_{min}) values compared to rounded to subrounded sands. Table 4.1 shows the relative effect of angularity, gradation and particle size on changes in e_{max} and e_{min} for natural sedimentary sands.

Clearly, of the three factors, viz., angularity, gradation and particle size, changes in e_{max} and e_{min} are most significantly controlled by the angularity characteristics of

Table 4.1

Limiting Void Ratios

Effect of angularity (a)	Cu = 1.2		Cu = 4		Cu = 10	
	R=0.17-0.3	R=0.17-0.7	R=0.17-0.3	R=0.17-0.7	R=0.17-0.3	R=0.17-0.7
Δe_{max}	0.55	0.76	0.42	0.60	0.37	0.53
Δe_{min}	0.33	0.41	0.27	0.34	0.22	0.29

Effect of Cu (b)	R=0.17		R=0.7	
	Cu=1.2-4	Cu=1-10	Cu=1.2-4	Cu=1-10
Δe_{max}	0.40	0.60	0.24	0.34
Δe_{min}	0.24	0.34	0.16	0.22

(a) & (b) - See Youd (1973)
(c.1) - Ishihara & Watanabe (1976)

Effect of particle size (c)	Cu=1-1.5	
	R=0.17	R=0.7 (c.1)
	D_{50}=0.10-2.0 mm	D_{50}=0.10-2.0 mm
Δe_{max}	0.26	0.16
Δe_{min}	0.05-0.08	0.04

sands. For example, for the variation of roundness index R from 0.17 to 0.7 (very angular to rounded) Δe_{max} for a sand with Cu = 1.2 is 0.76; 70% of this change in e_{max} being in the range of R = 0.17 to 0.30. Changes in particle gradation also produce corresponding changes in e_{max} and e_{min} depending on the particle angularity. For example, if Cu increases from 1.2 to 4.0, Δe_{max} is 0.4 and Δe_{min} is 0.24 for R = 0.17; the corresponding values for R = 0.7 being 0.24 and 0.16 respectively. Variation of particle size, D_{50} from 0.1 - 2 mm produces Δe_{max} = 0.26 for R = 0.17 and 0.16 for R = 0.7, the changes in e_{min} being considerably less.

Figure 4.1(b) shows the variation of (e_{max}-e_{min}) with D_{50}. Also shown in this figure are the results from Ishihara and Watanabe (1976). The results for glass beads show the maximum possible effect of size only (the glass beads being round and uniformly graded). For a given value of D_{50}, the difference between the results for glass beads, Kalpi sand and Fuji river sand fractions is perhaps a measure of the effect of angularity on (e_{max} - e_{min}). For Cu $\sim$ 1.1 to 1.2 the effect of angularity on (e_{max} - e_{min}) shown in Fig. 4.1(b) is comparable with the corresponding values deduced from Youd's data for R = 0.17 and R = 0.70.

Incidently the effect of D_{50} on (e_{max}-e_{min}) for rounded glass beads (D_{50} varying from 0.11 mm - 1.41 mm) is of the same order of magnitude as produced by changing Cu from 1.2 to 4.0 in case of rounded sands (R = 0.70). Tentatively it would thus seem reasonable to suggest that while angularity has overriding influence on (e_{max} - e_{min}), both gradation and particle size have

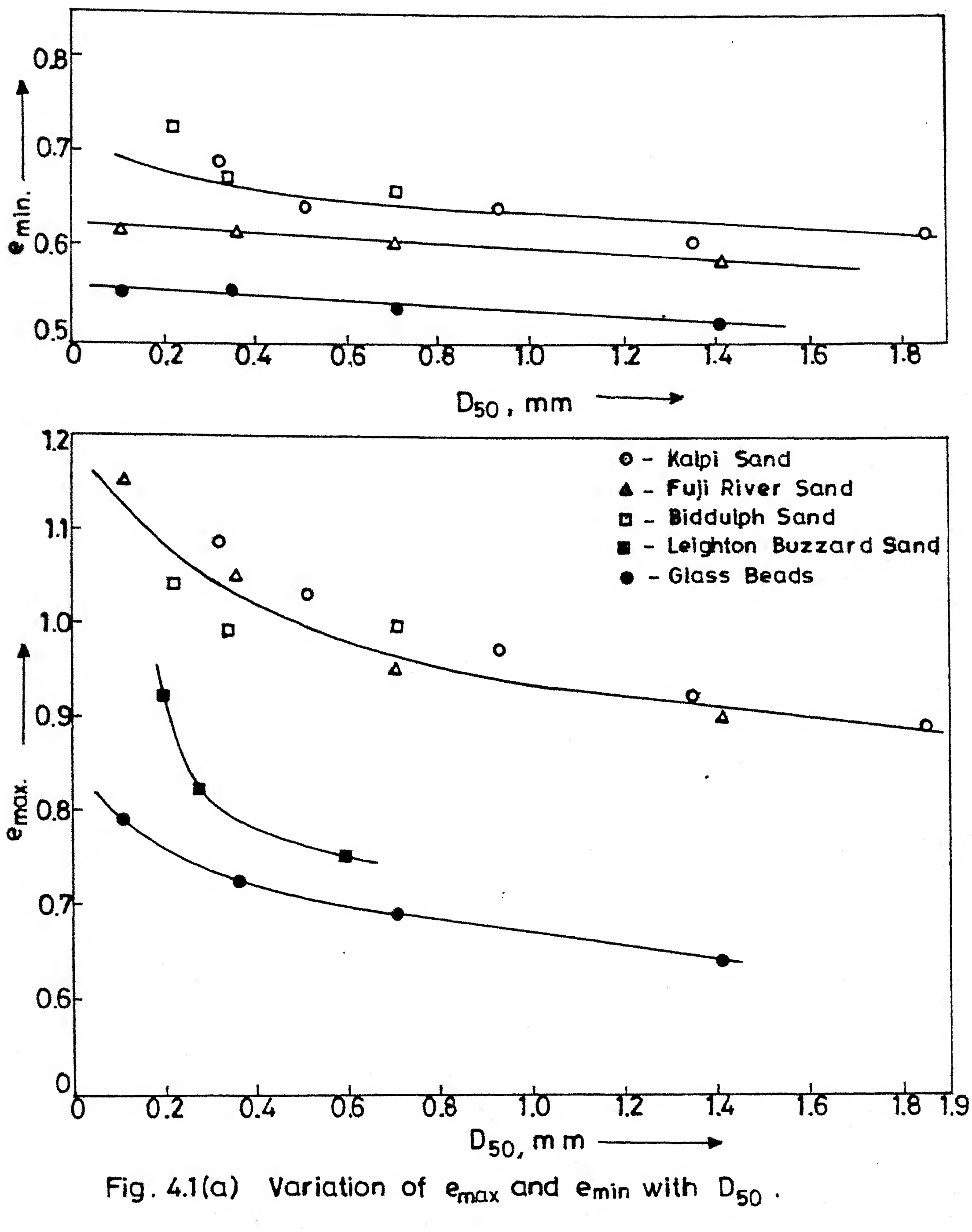

Fig. 4.1(a) Variation of e_{max} and e_{min} with D_{50} .

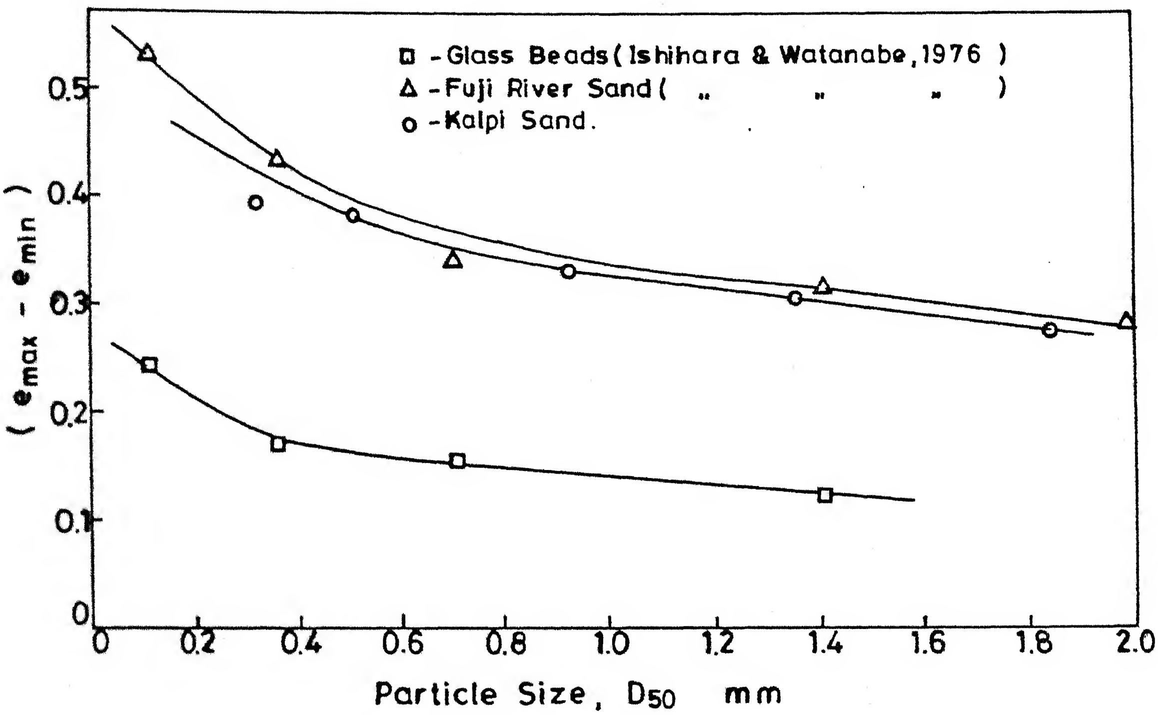

Fig. 4.1(b) Relationship Between (e_{max} - e_{min}) and D_{50}

comparable effects on its value. Also it may be stressed that any correlations between (e_{max} - e_{min}) and D_{50} or Cu alone, without any regard to angularity, are not valid. One will have to consider all the three factors while evaluating the maximum possible potential for volume change ((e_{max} - e_{min})).

4.3 Effect of Particle Size on Grain Packing

Variation of relative density with height of fall, for different fractions, during the raining technique adopted for sample preparation was shown in Fig. 3.2. Also superposed was the result for the natural Kalpi sand. It will be seen that poorly graded fine fractions would result in a highly loose sand deposit compared to the coarser fractions. The natural Kalpi sand containing all fractions (even grains bigger than 2.0 mm) tends to give results more or less similar to the coarser fractions. This may be due to the fact that majority of natural Kalpi sand comprises of coarser fractions (D_{50} = 1.06 mm). Also the well graded natural sand produces much denser packing compared to individual fractions, upto 10 cm height of fall. This has implication for preparing dense samples of uniformly graded sands. However, as pointed out earlier (3.3.4) the rate of pouring of sand has influence on relative density obtained for a given sand (Fig. 3.2(b)). Higher relative density samples were prepared by controlling the rate of fall in conjunction with the results shown in Fig.3.2(a).

4.4 Effect of Particle Size on Compressibility

Based on the oedometer test data already presented in Chapter 3, comparative behaviour of different fractions under stress is illustrated in Figs. 4.2(a) and (b). It will be seen (Fig. 4.2(a)) that for a given value of I_D, the finer the sand higher the compressibility. As the size increases, percentage compression at a given stress and relative density decreases. Furthermore, the maximum effect of particle size on compressibility is observed in samples with $I_D \leq 40\%$. Incidently most sedimentary sands in nature are deposited at relative density around 40% and therefore fine grained sandy deposits in the field are expected to experience higher compression under stress. Higher compressibility of loose fine fractions may be explained as due to two main factors:

(i) the finer fractions are much more angular than coarser fractions and

(ii) the number of interparticle contacts is small in loose state.

The number of contacts, as shown by Grivas and Harr (1980) is controlled by the value of porosity; the product of number of contacts and porosity being constant ($\approx$ 3). Fewer contacts with smaller area of contact for angular grains would result in higher interparticle stress which is likely to produce higher compression. Figure 4.2(b) shows the dependence of compression curves on relative density for a given value of grain size. For 0.212 mm grain size the compression of sand at a given

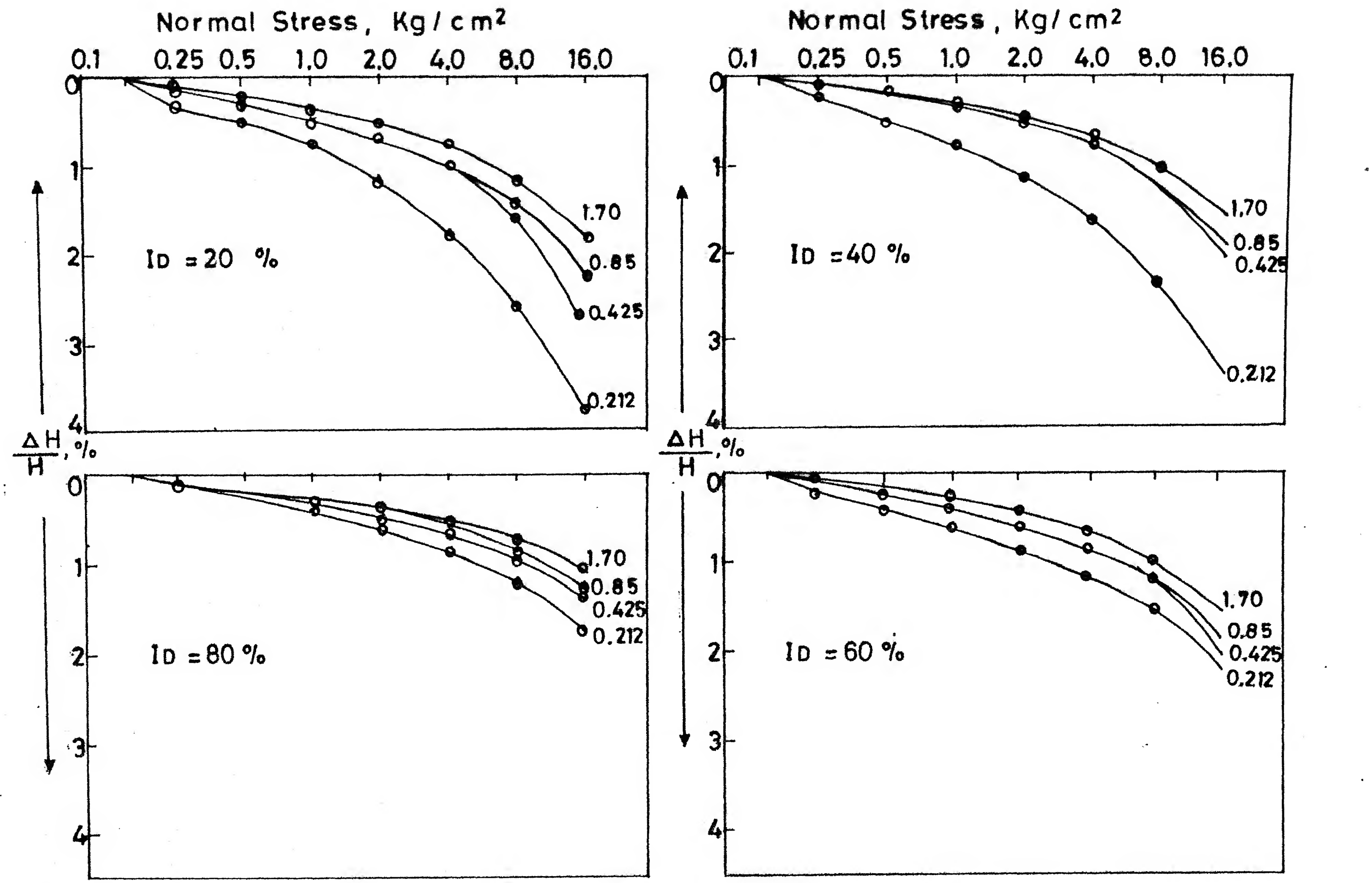

Fig. 4.2(a) Effect of Particle Size on Compressibility.

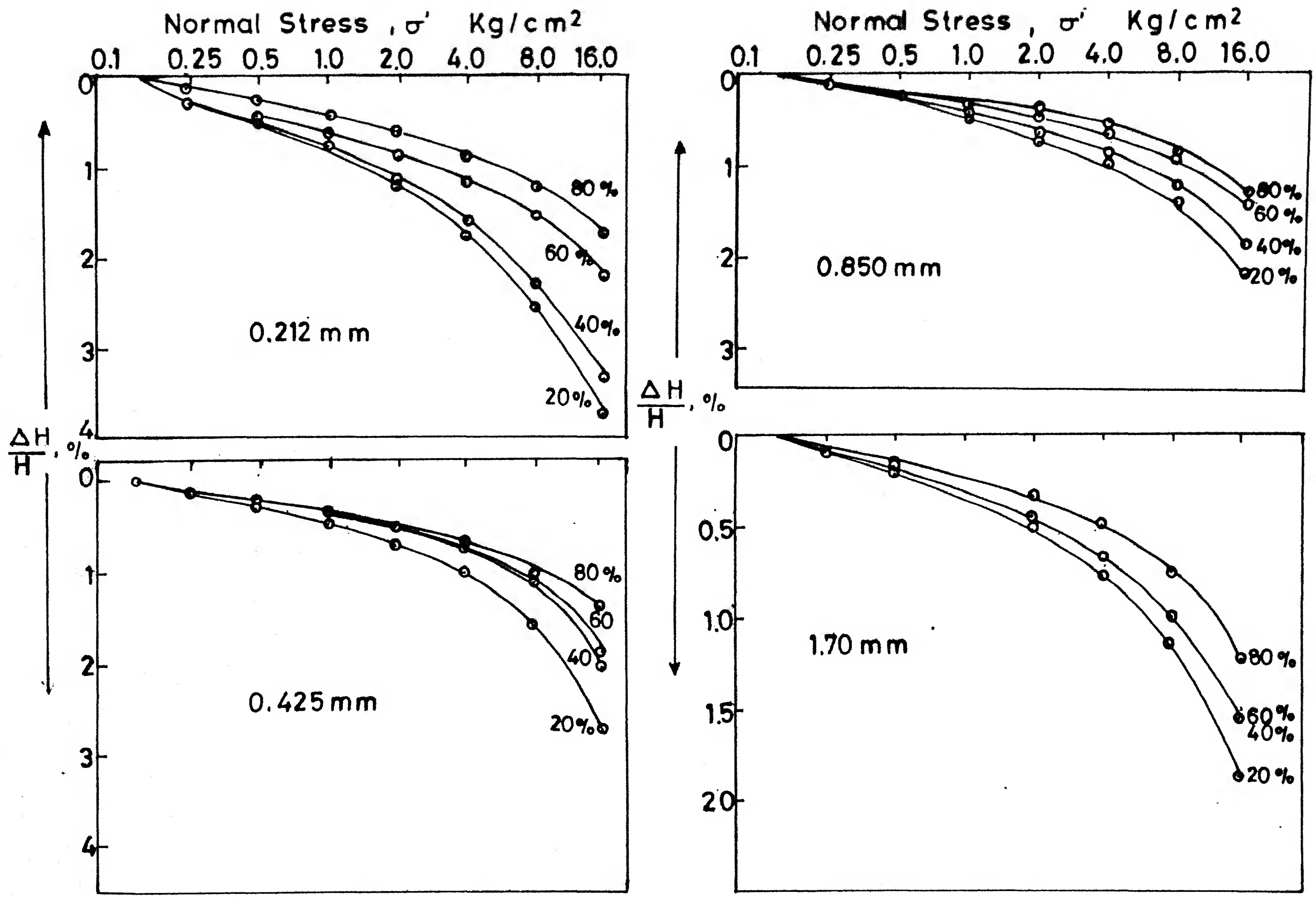

Fig. 4.2(b) Effect of Relative Density on Compressibility .

stress is very significantly affected by the value of relative density as compared to 1.70 mm grains (see Table 4.2). For example, at $\sigma' = 16$ kg/cm^2, $\delta(\Delta H/H)$ for a change in relative density from 20% to 80% is 0.02, 0.01, 0.009, 0.007 for 0.212, 0.425, 0.85 and 1.70 mm fractions respectively.

Table 4.2

Changes in Relative Density

Confining Stress	4 kg/cm^2			
Particle size, mm	0.212		1.70	
I_D, %	24.6	83	26	85
I_{DC}, %	34.4	87.3	31.8	88
I_{DF}, %	41.2	88	32.8	83.8

4.4.1 Constrained Modulus

The compressibility characteristics of different size fractions can be best illustrated through an examination of variation of constrained modulus with effective stress level, relative density and grain size.

Following Bellotti et al (1985) and Rahim (1989), constrained modulus may be expressed by the following relationship

$$M = m_o \times \sigma^{m_1} \times \exp(m_2 x I_D)$$

where m_o, m_1 and m_2 are constants. As brought out by Rahim, m_o,

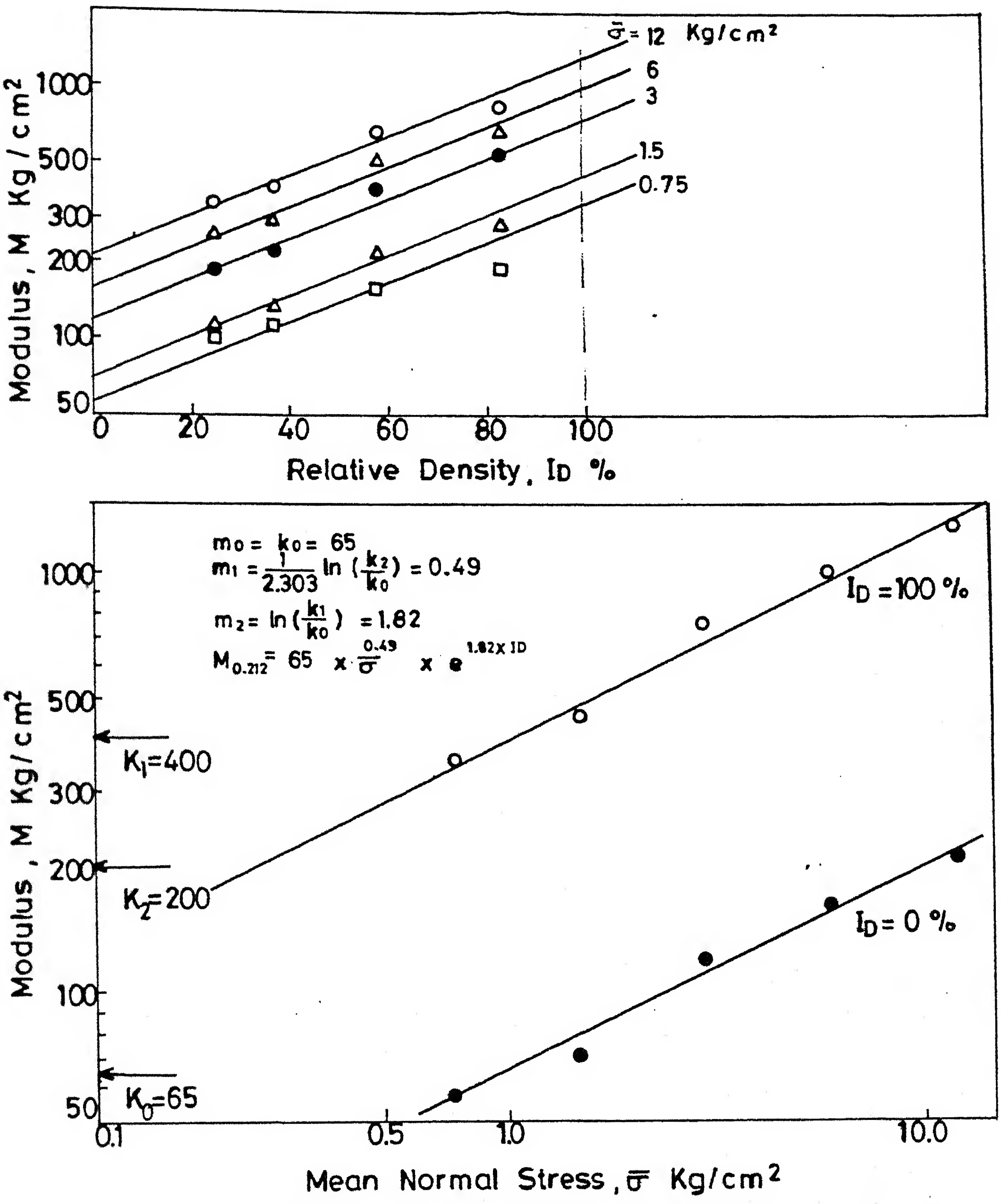

Fig. 4.3 Formulation of $M = f(\bar{\sigma}, I_D)$ for 0.212 mm Size.

(For details of procedure, see Rahim (1989))

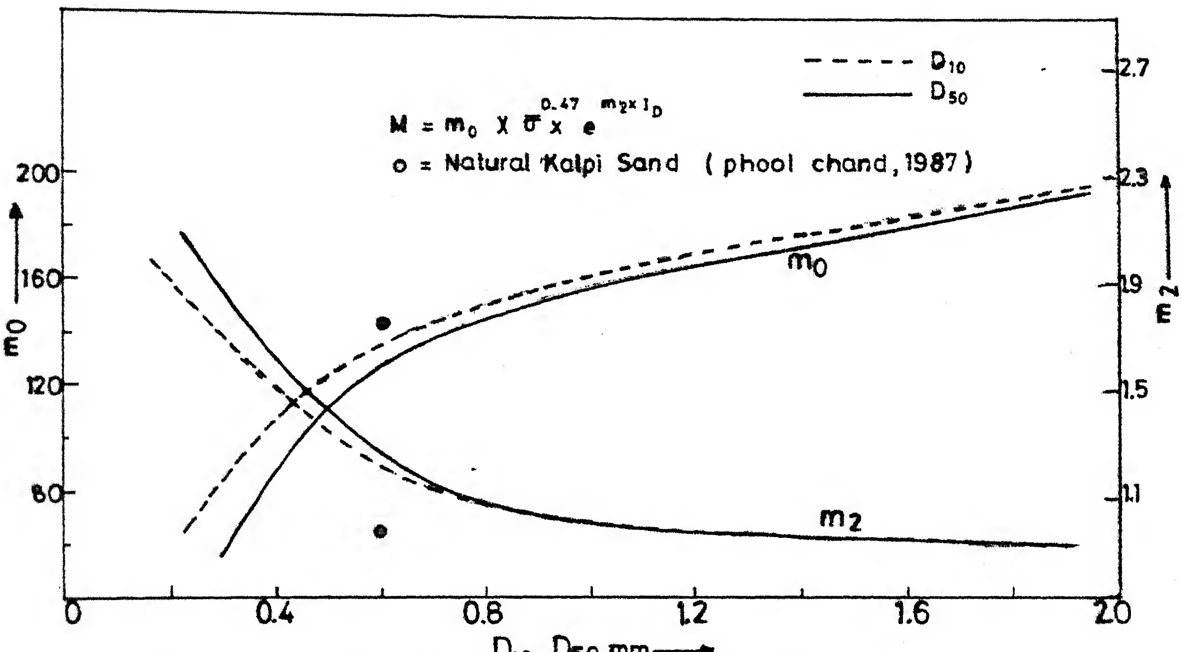

Fig. 4.4 Variation of m_0 & m_2 with D_{10} & D_{50}

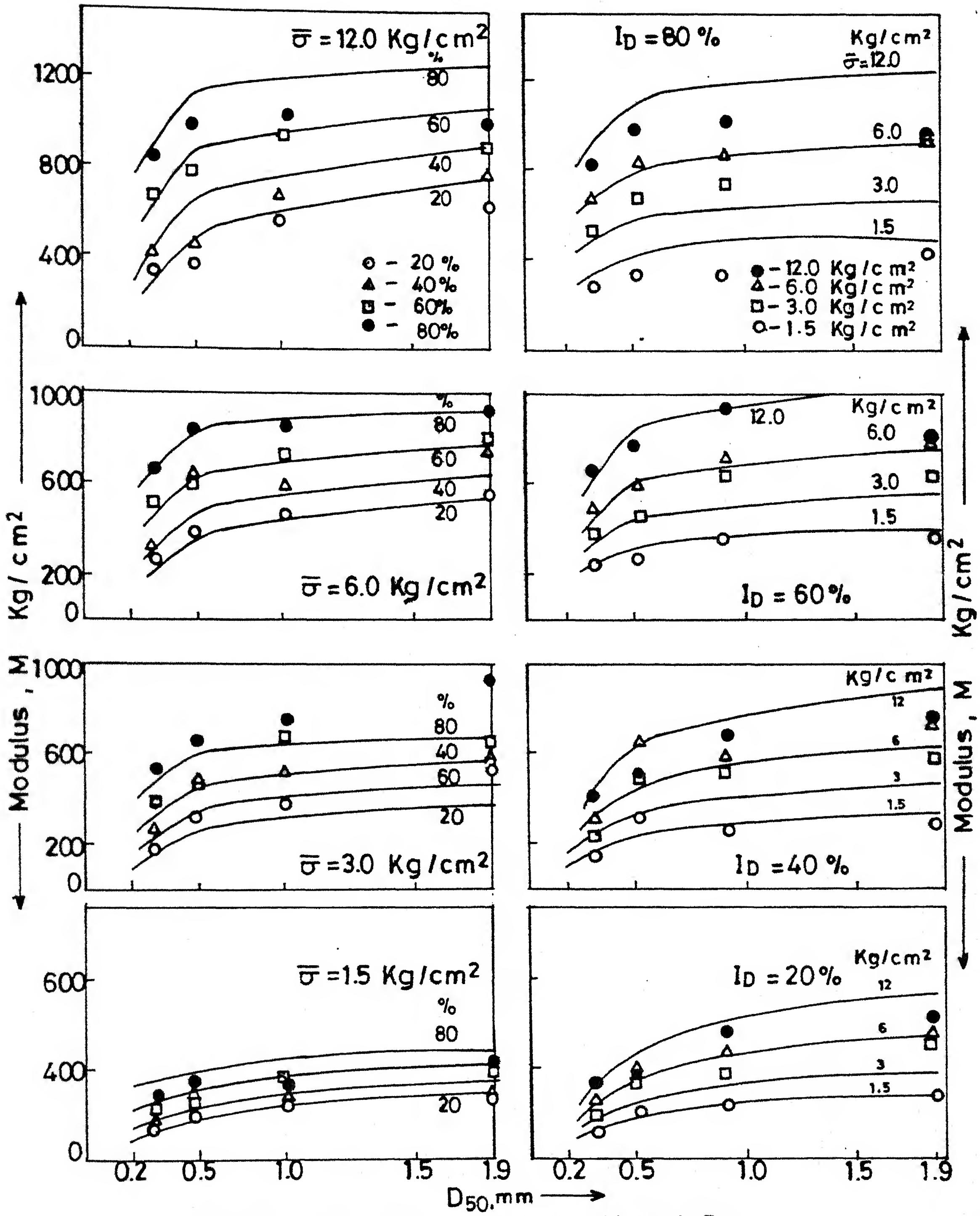

Fig. 4.5 Relationship Between M and D_{50}

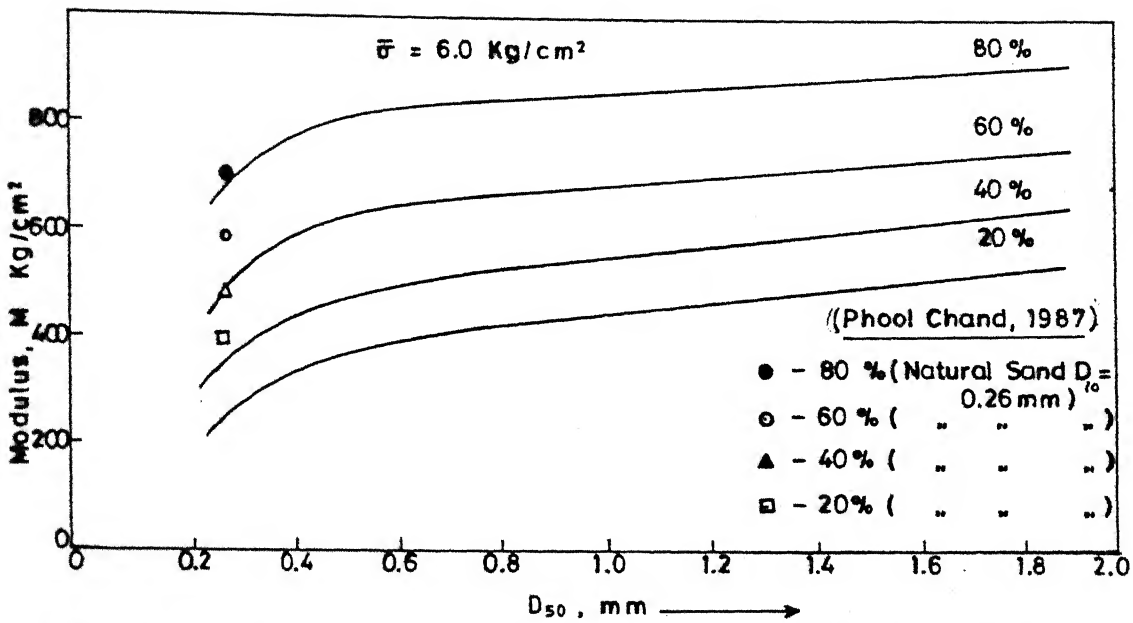

Fig.4.6 Comparison of Modulus of Natural Sand and Fractionated Sand.

m_1 and m_2 are governed by mineralogy and angularity characteristics of sands. On the basis of present investigation on different size fractions, dependence of m_o, m_1 and m_2 on particle size was investigated. Evaluation of m_o, m_1 and m_2 on the basis of compression test data is illustrated in Fig. 4.3. The test results suggest that while m_1 is independent of particle size, m_o and m_2 are functions of effective particle size. Fig. 4.4 shows the variation of m_o and m_2 with D_{10} and D_{50}. As will be seen from fig.4.4, modulus number, m_o, increases with increasing particle size, whereas m_2 - which reflects the contribution of relative density - decreases as the size increases. Rahim (1989) had concluded that m_o is strongly controlled by mineral composition (high value for incompressible strong minerals (Quartz),low value for carbonates and micaceous sands) and m_2 is significantly governed by angularity characteristics of sands. He showed some variation of m_1 with compressibility - from 0.57 for high compressibility to 0.37 for low compressibility sands. In the present study the value of $m_1 = 0.47$ fits very well with the value of 0.42 recommended by Rahim for sands of moderate compressibility. Clearly m_1 appears to be independent of D_{50} for a sand of given mineralogy and angularity. Interestingly, increase of m_o with particle size is analogous to decrease in m_o with compressibility. Increase in particle size results in reduced compressibility. Thus m_o seems to be affected by both mineralogy and particle size.

Rahim (1989) pointed out the dependence of m_2 on angularity characteristics of sands i.e., higher the angularity, smaller m_2, meaning thereby lesser effect on changes in relative density alone. The present results,using different size fractions having comparable angularity bring out the dependence of m_2 on particle size. For a given angularity m_2 is shown to decrease as particle size increases. These two findings suggest that for the same particle size (D_{10}), m_2 is higher for sands of low angularity (low compressibility) compared to sands of high angularity (high compressibility). Therefore in case of natural sands, consisting of grains of different sizes and angularity, these two effects of size and angularity on m_2 will produce the resultant value of m_2 for the sand which will reflect the response of the sands to the changes in relative density. Ofcourse mineralogy variations among different fractions of a sand (if present) will also contribute to the final value of m_2.

Using the appropriate values of m_o, m_1 and m_2 (from Fig. 4.4) the relationship ($M = m_o \times \bar{\sigma}^{m_1} \times \exp(m_2 \times I_D)$) the variation of constrained modulus with particle size for different confining stress and relative density, is shown in Fig. 4.5. Also compared are the experimental results. It will be seen that the equation ($M = m_o \times \bar{\sigma}^{m_1} \times \exp(m_2 \times I_D)$) predicts the value of modulus reasonably well.

Though in Fig. 4.5 the values of modulus were compared with D_{50} as a measure of particle size,it would seem more appropriate to use D_{10} (effective diameter) instead of D_{50} for comparison of compressibility of different natural sands of varying gradation.

Burmister (1962) shows systematic reduction in compressibility with increasing D_{10} values, for eight sands of varying gradation; the D_{50} values for these sands donot systematically follow the same trend as D_{10}. Furthermore compressibility, like permeability, of sands is affected by the percentage of fines present and the effective diameter D_{10} properly represents the finer fractions in sands. As is well known D_{10} is also commonly used to correlate with coefficient of permeability (for example $K = 100\ D_{10}^2$). In case of uniformly graded sands, however (as in the present study) D_{10} or D_{50} are practically identical and this is the reason why in Fig. 4.5, D_{50} was used. The trends and relative comparison with experimental data will remain essentially same if D_{10} instead of D_{50} was used in Fig.4.5.

In Fig.4.6 the variation of modulus for different size fractions with D_{10} has been depicted at $\bar{\sigma} = 6.0\ kg/cm^2$ for I_D values varying from 20% to 80%. This was done so as to be able to compare these results with those predicted for natural Kalpi sand. Phool Chand (1987) gives the following relationship for the natural Kalpi sand

$$M = 142 \times \bar{\sigma}^{0.46} \times \exp(0.94 \times I_D)$$

It will be seen that for the same D_{10} value, the poorly graded fractions are more compressible (low M value) almost upto 60% relative density. Only for $I_D \geq 80\%$ the well graded sand and uniformly graded fraction seem to give similar values of modulus which is perhaps to be expected, since for $I_D \geq 80\%$ sands would be approaching densest possible packing.

The loading-unloading comparison data shown in Fig. 3.3(a)

to Fig. 3.3(d) suggest that:

(i) For a given size, relative density appears to have no effect on slope of the unloading-reloading curve, however as stress level increases beyond 1 kg/cm^2, there is a general tendency for increase in slope of the unloading curve.

(ii) For a given relative density, particle size seems to have practically no effect on the slope of the unloading-reloading curve.

(iii) The difference in unloading-reloading modulus between values of finer fraction (D_{10} = 0.23 and I_D = 20% at $\bar{\sigma}$ = 12 kg/cm^2) and the coarser fraction (D_{10} = 1.73 and I_D = 80% at $\bar{\sigma}$ = 12 kg/cm^2) is of the order of 10-15% only. For these two extremes at an average stress of 12 kg/cm^2, the ratio of unload-reload modulus to normally consolidated value varies from 20 to 6 for fine to coarse fractions respectively. This ratio shows decrease with increase in past maximum pressure.

There is a clear need for more extensive study to evaluate exact relationship between modulii ratios and past maximum pressure for sands of varying compressibility. Phool Chand (1987) has suggested that past maximum pressure has effects on the ratios of these modulii for highly compressible sands only. The present study on uniformly graded fractions of moderate compressibility does not indicate significant effect of particle size and relative density on the unload-reload modulus for a given past maximum pressure. As pointed out earlier, the maximum effect of compressibility on unload - reload modulus for these uniformly graded varying size fractions is of the order of 10-15%.

4.5 Effect of Particle Size on Shear Strength

Based on the direct shear test data presented in Chapter 3, comparative behaviour of 0.212 mm and 1.70 mm size sand is illustrated in Fig. 4.7(a) to Fig. 4.7(e). The following observations have been made on the stress strain behaviour of these sands:

1(a). For same relative density and normal stress peak strength increases with the particle size.

(b). For same relative density the difference between peak strength values for D_{50} = 0.32 mm and D_{50} = 1.85 mm increases with increasing confining stress (upto 4.0 kg/cm^2).

2(a). For the same particle size, initial shear stiffness increases with the relative density and normal stress.

(b). For the same relative density and normal stress, initial shear stiffness increases with size.

3(a). Peaked behaviour, as expected, increases with relative density.

(b). For the given relative density and normal stress, peaked behaviour increases with grain size.

4(a). Response to volume change during shear is significantly controlled by the particle size (see Table 4.2).

(b). In the case of sand fraction with D_{50} = 0.32 mm, at same normal stress, volume change during shear gradually changes from positive (compression) to negative (expansion) as relative density increases from 20% to

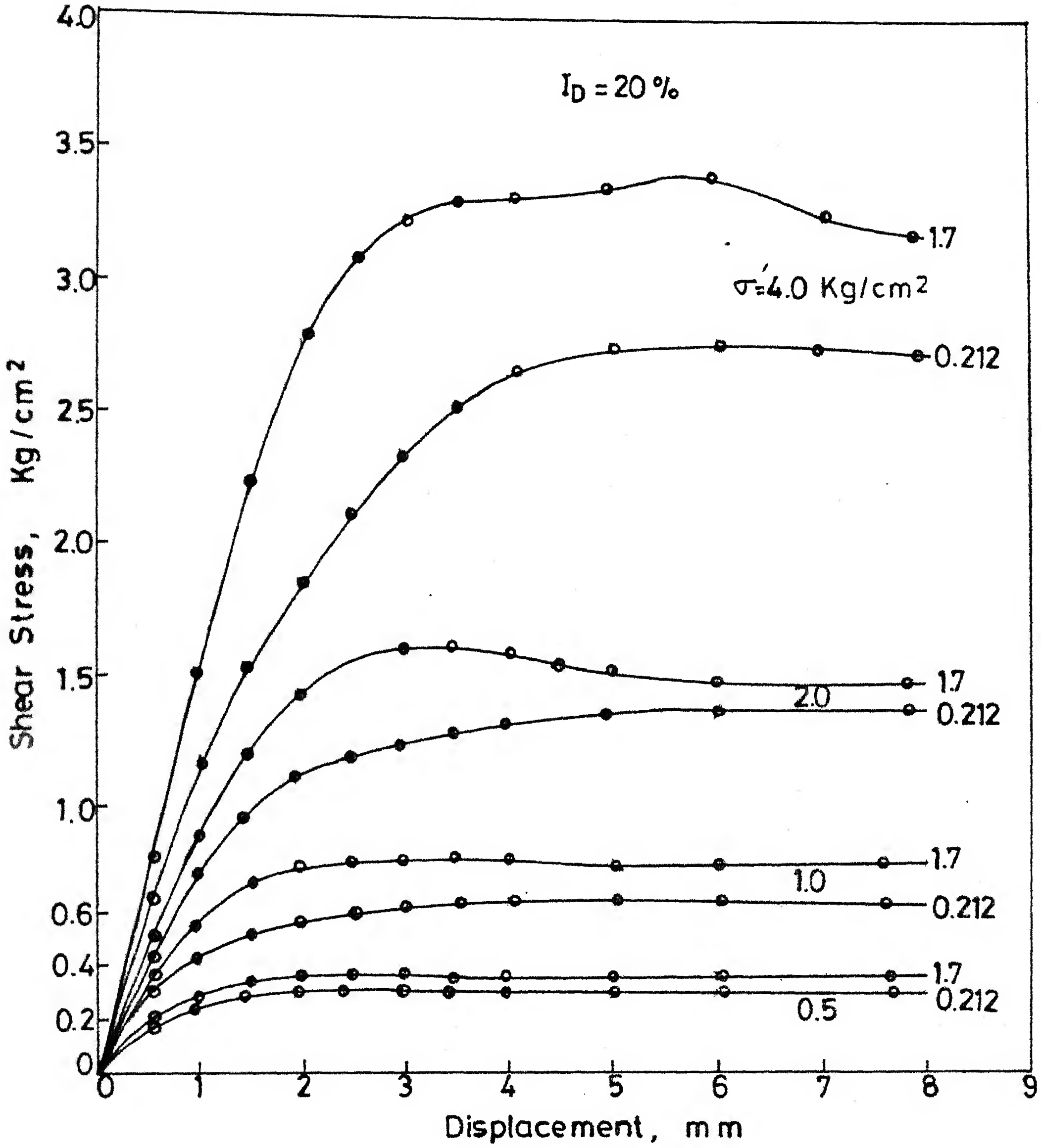

Fig. 4.7(a) Comparison of Effect of Size in 0.212 & 1.7 mm Sand.

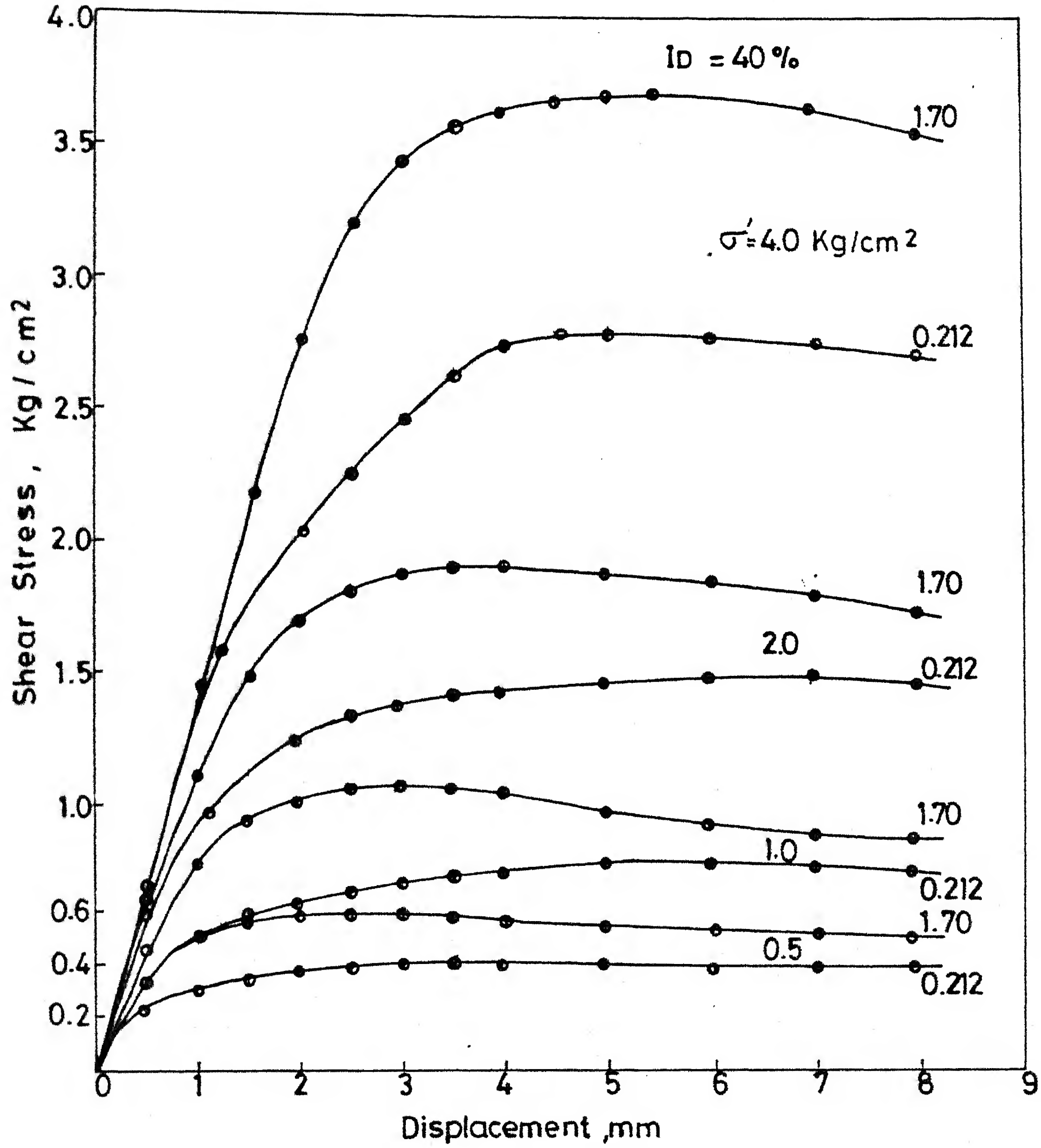

Fig. 4.7 (b) Comparison of Effect of Size in 0.212 & 1.70 mm Sand.

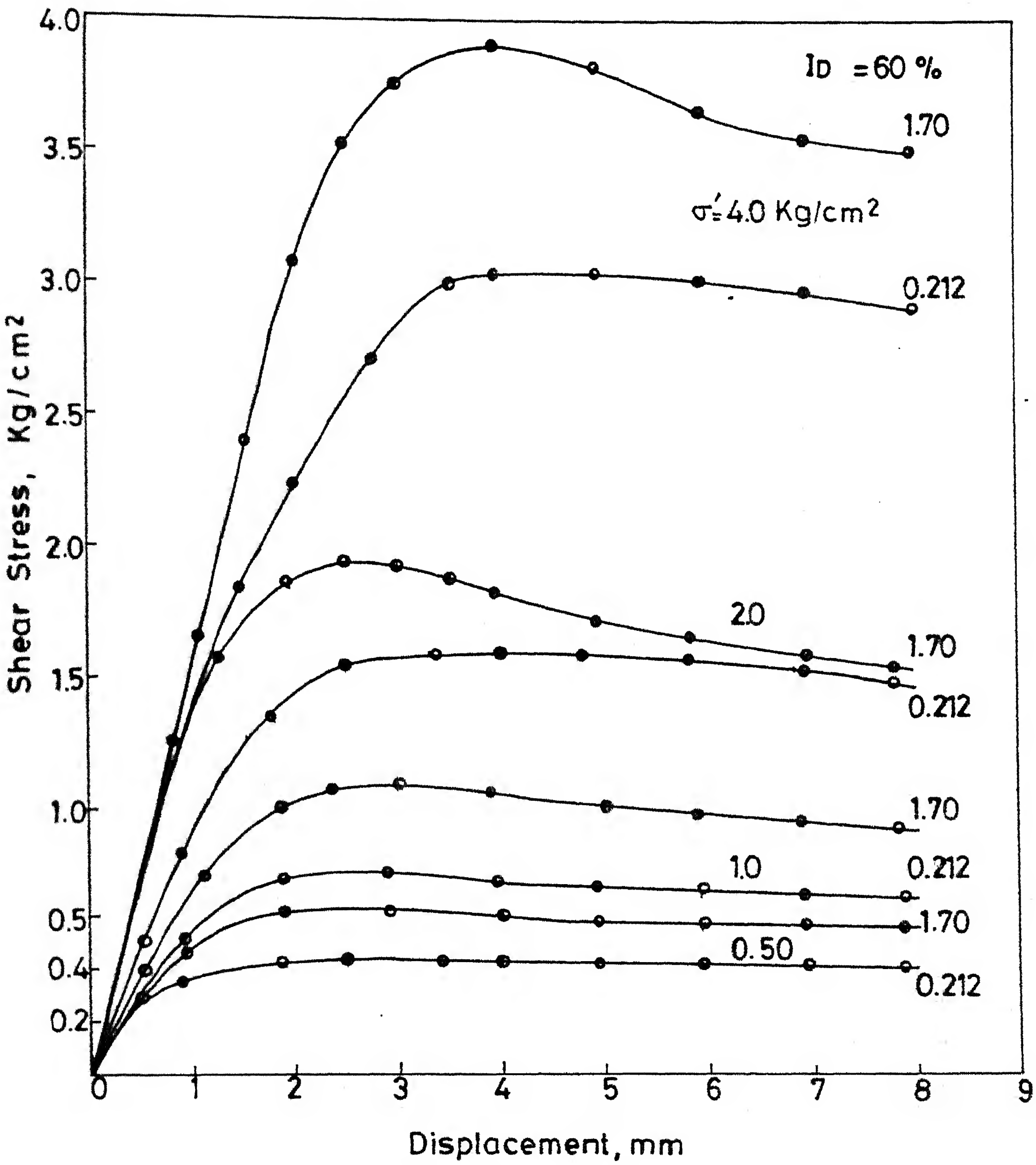

Fig.4.7(c)Comparison of Effect of Size in 0.212 & 1.70 mm Sand.

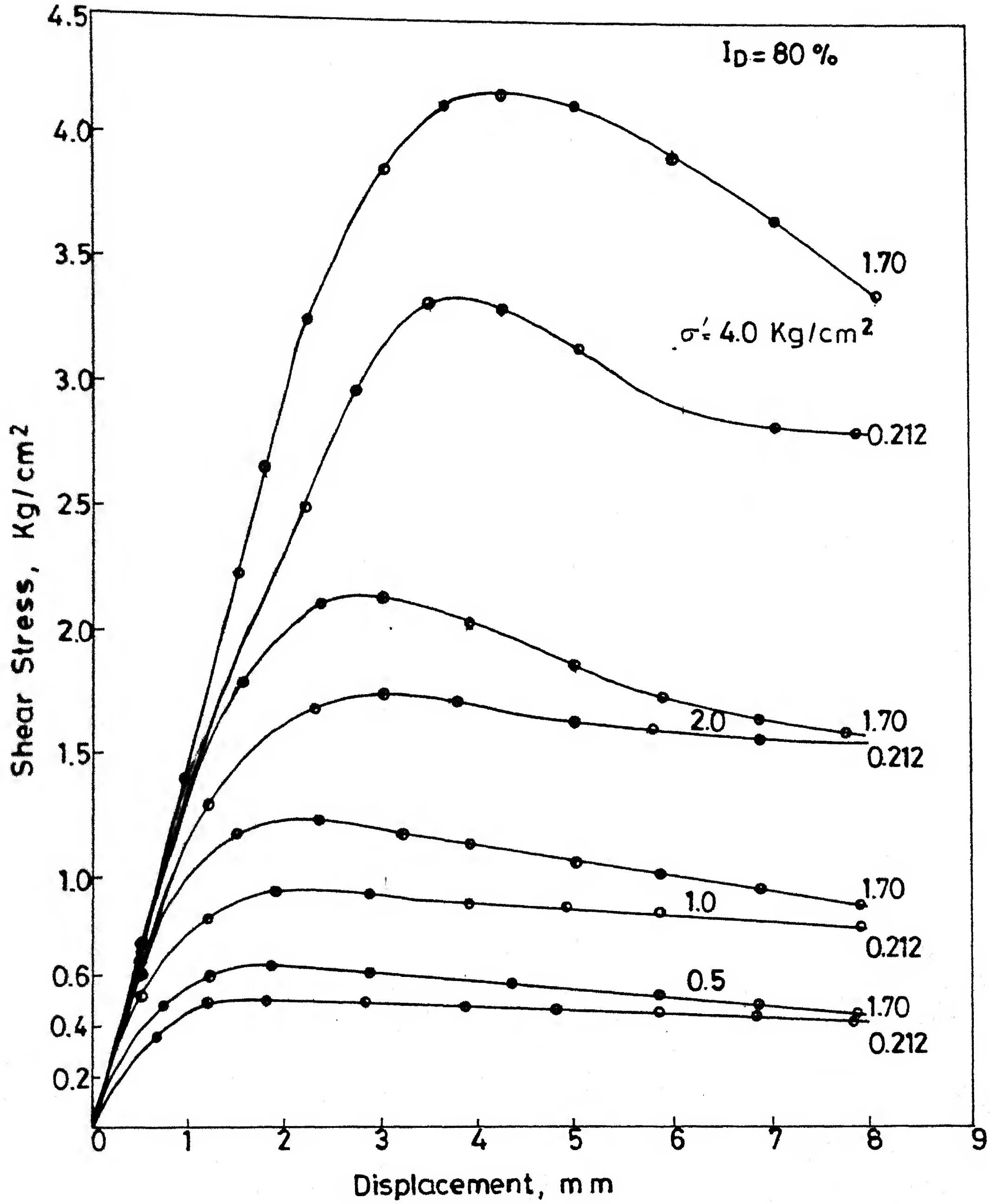

Fig. 4.7 (d) Comparison of Effect of Size in 0.212 & 1.70 mm Sand.

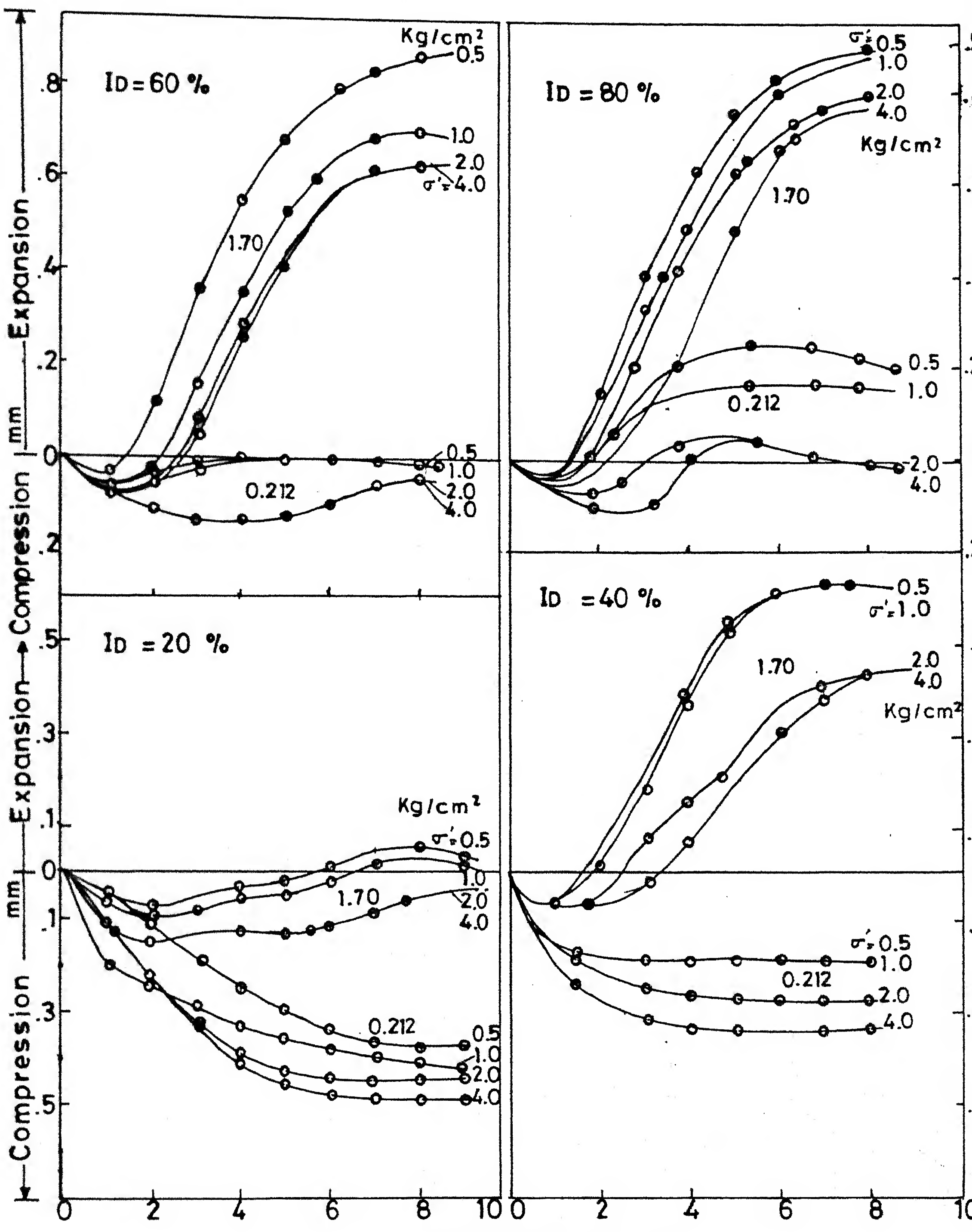

Fig. 4.7 (e) Comparison of Dilantancy in 0.212 & 1.70 mm Sand .

80%; however, in case of sand fraction with D_{50} = 1.85 mm, the material expands during shear for all values of relative density investigated.

(c) As expected, in case of finer fraction, compression during shear increases with normal stress, whereas in case of coarser fractions, the amount of expansion reduces as the confining stress increases.

The above mentioned observations lead to the following conclusions:

1. The shear strength increases with particle size.
2. Initial shear stiffness increases with particle size.
3. Dilatancy increases with particle size.

As discussed above, the peak shear strength - normal stress relationship is expected to be controlled by the particle size depending on the magnitude of relative density. In Fig. 4.10 are shown results of experimental data along with prediction made by a power law equation of the form

$$\tau = \alpha \times \sigma'^{\beta^*} \times \exp(\gamma \times I_D)$$

as suggested by Phool Chand (1987); coefficients α , β^* and γ in the above equation are obtained by a graphical procedure shown in Fig. 4.8. While β^* and γ are found to remain constant with size, α varies with D_{50} as shown in Fig. 4.9. It will be seen that except for I_D = 20%, the experimental results tend to agree reasonably well with the predicted values (Fig. 4.10). In case of I_D = 20%, the various size fractions shown either decrease in volume or no change in volume during shear and the failure

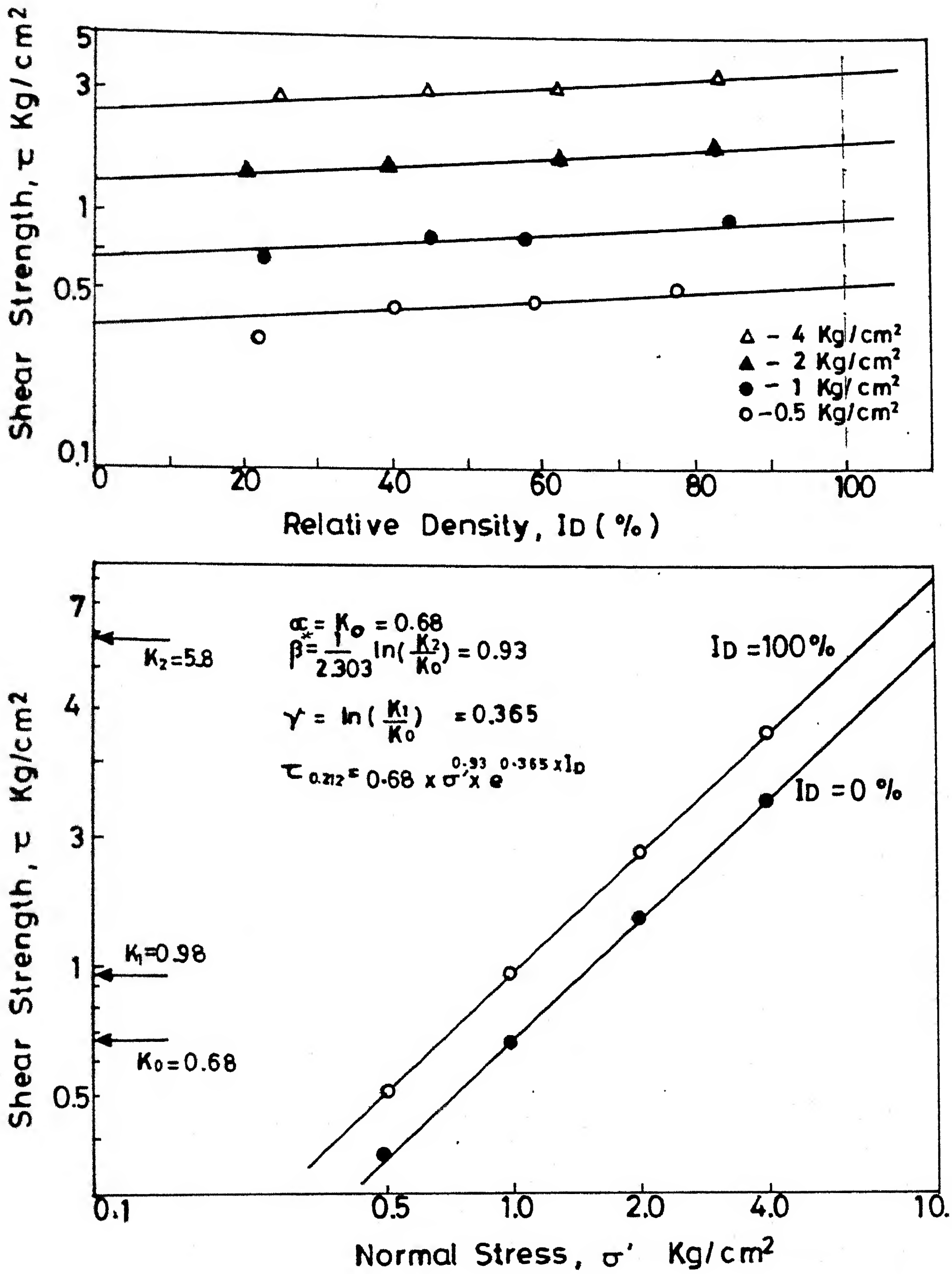

Fig.4.8 Formulation of $\tau = f(\sigma', I_D)$ for 0.212 mm Size.

(For details of Procedure, see Phool chand (1987))

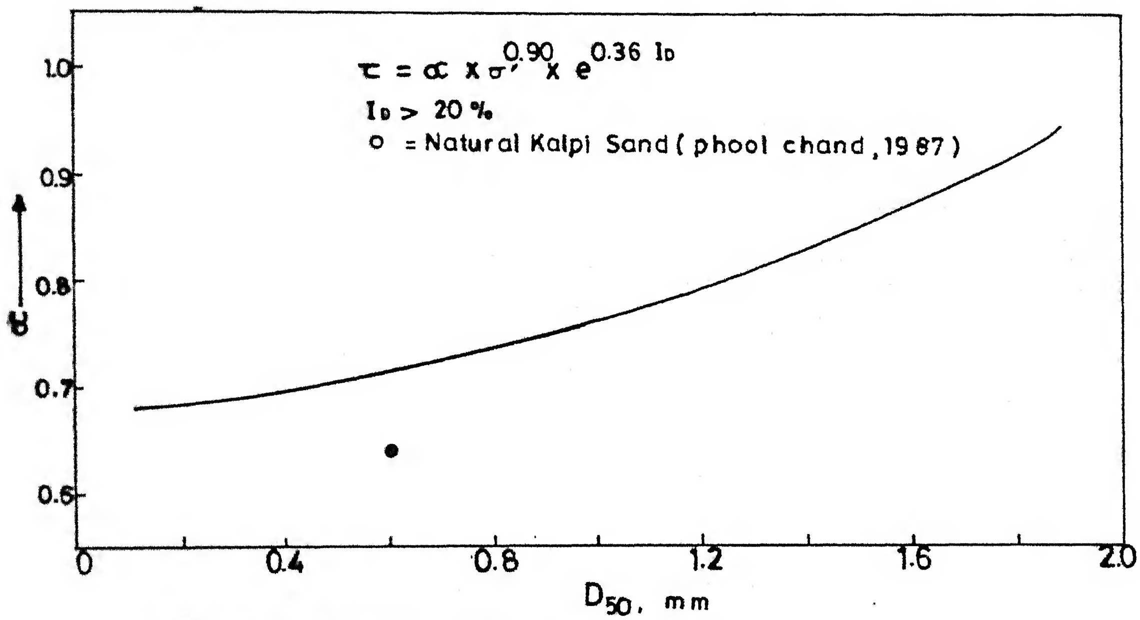

Fig.4.9 Variation of α with D_{50}

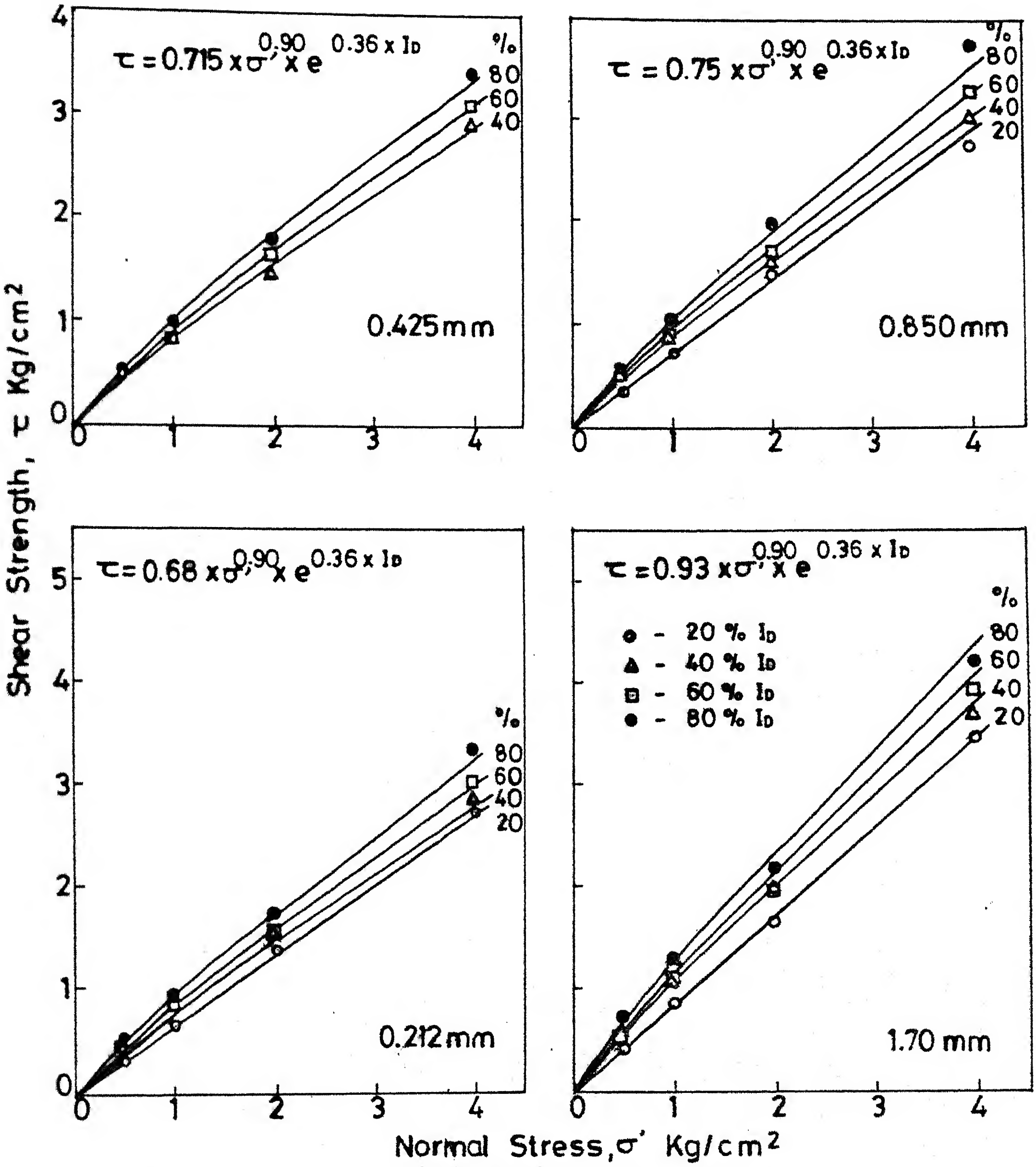

Fig. 4.10 Shear Strength vs Normal Stress Envelope .

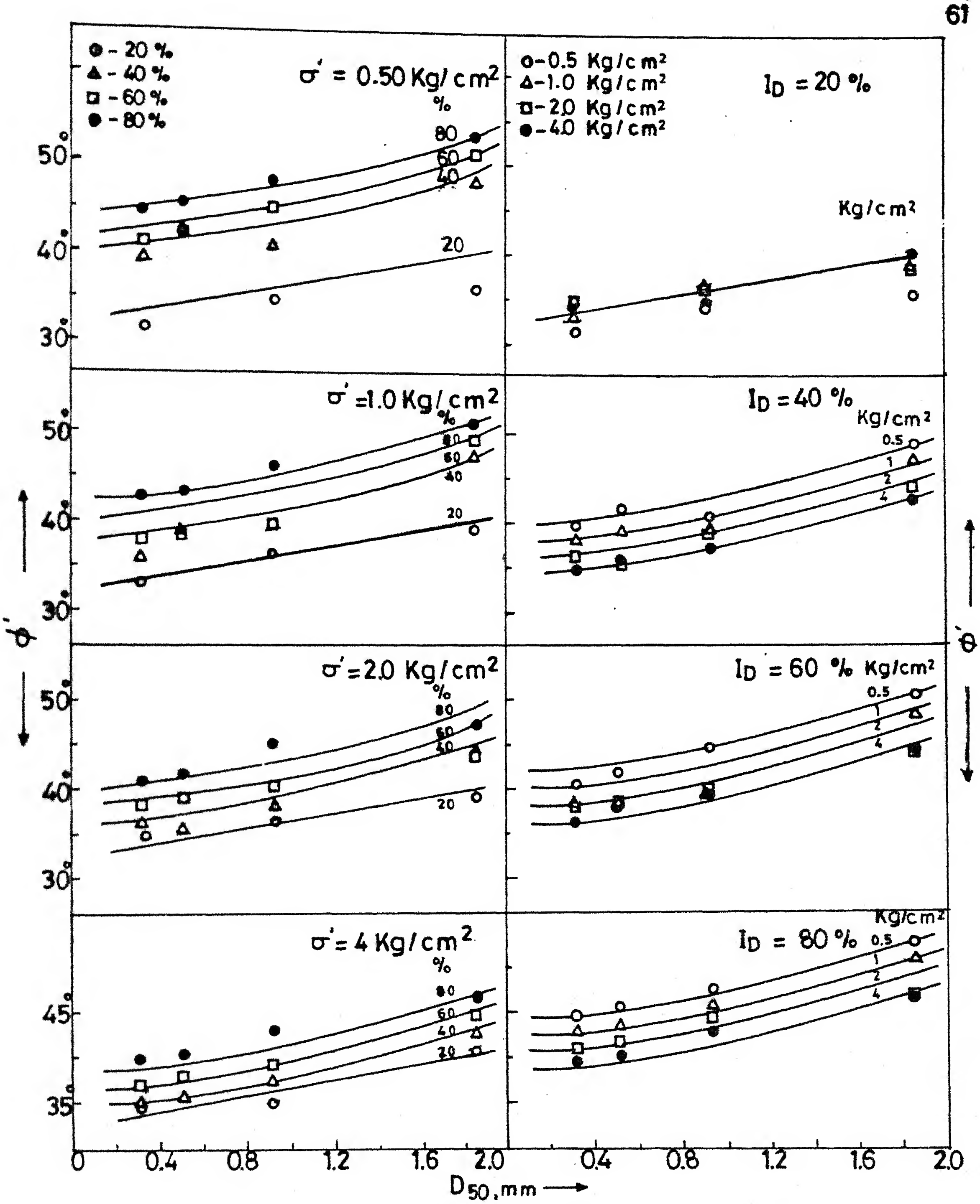

Fig. 4.11 Variation of ϕ' with D_{50}

envelope observed for all fractions is best represented by Coulomb's law.

Figure 4.11 shows variation of equivalent angle of shearing resistance (secant value) with D_{50}. It will be seen that predicted and experimental values show reasonably good agreement. Angle of shearing resistance, at a given normal stress and relative density increases with particle size.

At this stage it would be interesting to compare the curvature in both $\tau - \sigma'$ and $M - \sigma'$ plots. Typical results for I_D = 60% are shown in Fig. 4.12. It will be seen that modulus envelope shows much pronounced curvature than the failure envelope. Also shown in the same figure are the results predicted for the natural Kalpi sand. The power law equation with appropriate values of α, β and γ were used.

As discussed earlier D_{10} is more relevant size parameter for compressibility and D_{50} is the relevant size parameter for shear strength. For the same D_{50}, as expected, well graded natural Kalpi sand gives higher strength than the uniformly graded fraction of the same sand. However, for the same D_{10}, the uniformly graded fraction of Kalpi sand is more compressible (low modulus) than the well graded natural Kalpi sand.

4.6 Application

One of the direct consequences of influence of particle size on mechanical behaviour of sands is the dependence of resistance to penetration in case of cone and SPT, on the D_{50} values. Skempton (1986) brought out this interdependence after

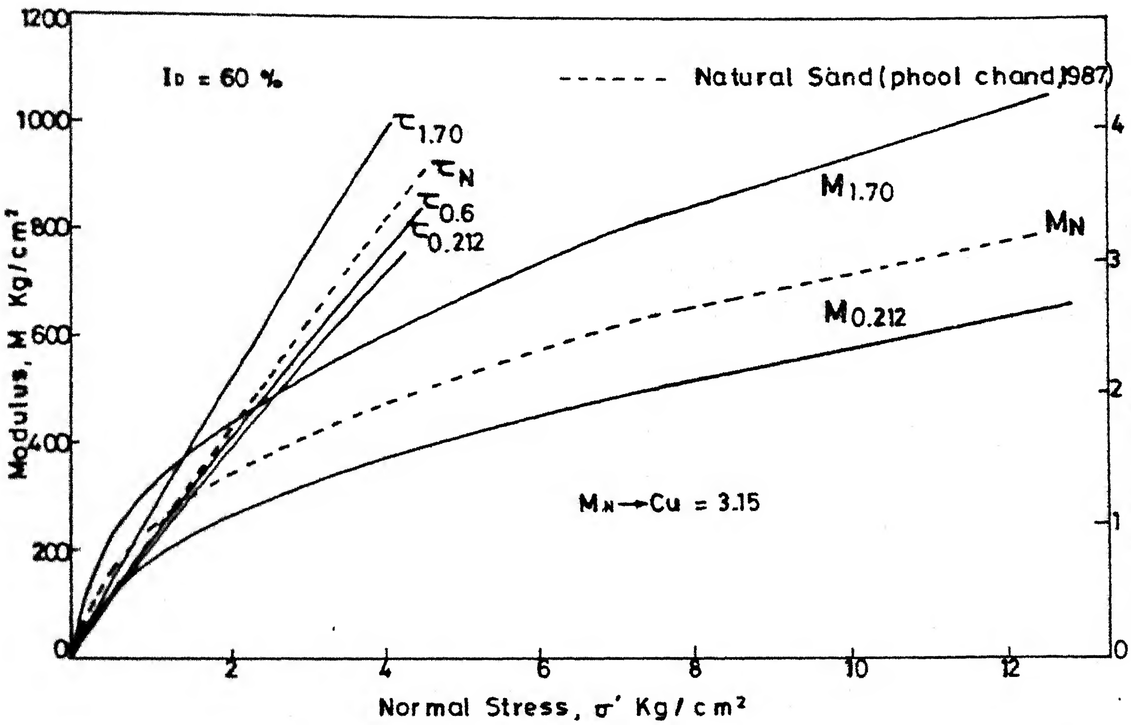

Fig. 4.12 Comparison of Curvature (M vs σ' and τ vs. σ')

reviewing the available field records for a variety of sands. The value of bearing capacity factor N_q was predicted from the following rela$tionship.

$$N_q = \tan^2 (45+\Phi'/2) \exp((\Pi - 2\beta) \tan \Phi')$$

as proposed by Janbu and Senneset (1974)

The values of β were assigned to sand fractions on the basis of their observed compressibility so as to choose the appropriate mode of failure. The appropriate I_D values under σ' were computed using consolidation test data for I_D = 40% (Fig. 3.3(a) to Fig. 3.3(d)). The corresponding values of Φ' for different sizes were obtained by using the power law equation and Fig. 4.9.The cone resistance,q_c was found out for each fractions under varying normal stresses by the equation

$$\frac{q_c}{\sigma'} = N_q$$

From Robertson and Campanella (1983) the values of q_c/N for different D_{50} values were obtained, are given in Table 4.3:

Table 4.3

q_c/N Values

D_{50},mm	0.32	0.51	0.92	1.85
q_c/N	5.4	6.0	7.7	10

The q_c values were thus converted to N values and by using the relevant I_D, values of N/I_D^2 were obtained.

Figure 4.13 shows the N/I_D^2 versus σ' relationship. Also

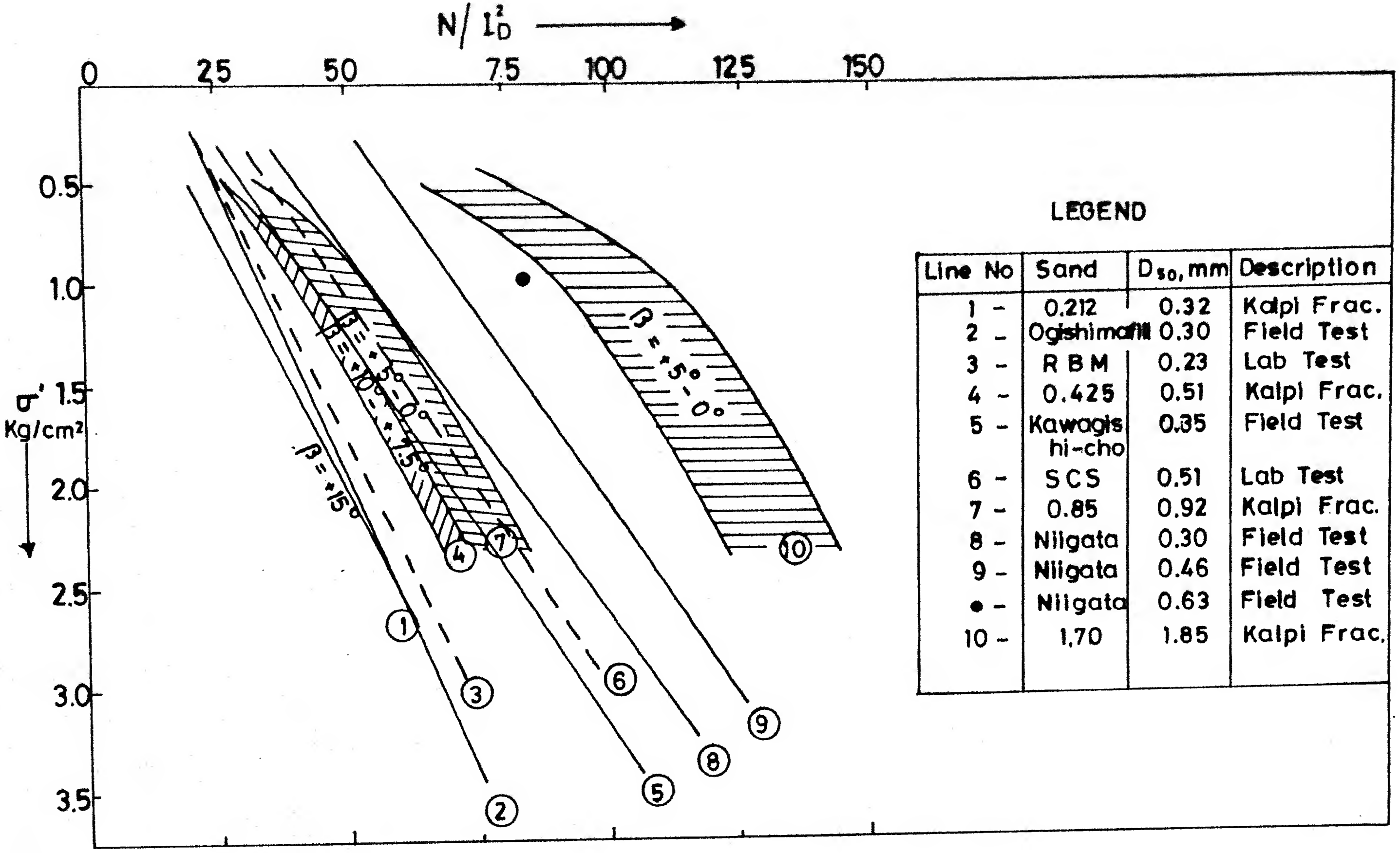

Fig. 4.13 Effect of Overburden Pressure .

shown are the results reported by Skempton (1986). In general, at a given σ', N/I_D^2 increases as D_{50} increases. The values of β used in the computation are indicated in the figure. The laboratory results quoted by Skempton (1986) seem to fit well with the predicted behaviour. The higher field values are clearly the result of ageing as pointed out by Skempton.

It must be noted that the predictions are indeed tentative, since the results are very sensitive to β values. For a more accurate prediction, the relationship between soil compressibility and values needs to be established. Perhaps Vesic's rigidity index (Vesic, 1970) may be used to relate compressibility factors to the value which governs the mode of failure.

CHAPTER 5

CONCLUSIONS

On the basis of the detailed investigations undertaken in the present study to examine the effect of particle size on volume decrease potential, compressibility and shear strength behaviour of sands, the following conclusions may be made:

1. It is observed that, there is a pronounced effect of particle size on e_{max}, but the effect on e_{min} is minimal. e_{max} decreases with particle size, thereby producing a decrease in volume change potential ($e_{max} - e_{min}$) as the D_{50} increases. For evaluating the limiting void ratios, the effect of angularity, gradation and particle size must be taken into account.
2. It is brought out that the poorly graded fine fractions would result in a highly loose sand packing compared to the coarser fractions. Gradation has more effect on relative density at small heights of fall, but particle size has an overriding effect at large heights of fall during raining technique (Fig. 3.2(a)).
3. It is observed that the compressibility decreases as the particle size increases, for a given relative density (Fig. 4.2(a)). The compression of the smaller fraction is very significantly affected by the value of relative density compared to that of larger fraction.
4. The power law proposed by Belloti et al (1985) and further modified by Rahim (1989) was found suitable for sand fractions of different particle sizes. The test results

suggest that while modulus parameter m_1 is independent of particle size, m_o and m_2 vary with effective particle size (Fig. 4.4). The agreement between the experimental and the predicted values of the constrained modulus is quite satisfactory.

5. It is suggested that, for comparing the modulii of well graded sand and uniformly graded sand, D_{10} is the relevant particle size parameter.

6. The particle size and relative density seem to have very little effect on the slope of the unloading-reloading curve. Past maximum pressure is the main controlling factor. The ratio of unload-reload modulus to the normally consolidated value varies from 20 to 6 for fine to coarse fractions and the ratio decreases with increase in past maximum pressure.

7. The shear strength, initial shear stiffness and dilatancy increases with particle size. In case of strength characteristics, D_{50} is the relevant particle size parameter.

8. The power law equation proposed by Phool Chand (1987) was found to be in good agreement with the test data for moderate to dense packing. The coefficients β and γ are found as remain constant with particle size, α increases with D_{50} (Fig. 4.9). However, for loose packing, the failure envelope for all fractions is best represented by Coulomb's law.

9. It is observed that the equivalent angle of shearing resistance increases with particle size at given normal

stress and relative density. The agreement between the predicted and the experimental values of ϕ' is quite satisfactory (Fig. 4.11).

10. The modulus envelope shows a much pronounced curvature than the failure envelope (Fig. 4.12). The predicted values of modulus and shear strength of natural Kalpi sand are greater than those of uniformly graded fractionated sand with appropriate particle size (i.e. D_{10} for modulus and D_{50} for shear strength).

11. By comparing the predicted SPT resistance of the Kalpi sand fractions with the available information in the literature, it is observed that the influence of the particle size on SPT values is pronounced. The gradation and ageing of sand also contributes to SPT value at a given effective overburden stress.

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